Cooperative and Non-cooperative Models for slotted-Aloha type MAC protocols

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1. INTRODUCTION

Aloha [1] and its slotted variation [5] have been widely deployed as a medium access control (MAC) protocol for different communication networks. Slotted-Aloha type MAC protocols don't perform carrier sensing and synchronize the transmissions into time-slots. These protocols are suitable for controlling multiple accesses when nodes cannot sense each other. For example, two nearby wireless LANs [2] may compete for shared media, where nodes in one WLAN don't notice the existence of the other WLAN. Recent development of wireless and sensor networks urges us to re-investigate slotted-Aloha type MAC, and to design its variations for these new trends.

Early researches had focused on the stability control [6, 4] aspect of the protocol. One recent work [3] discussed the stability of slotted-Aloha with selfish user behaviors and perfect information. In this paper, we focus on the performance issues of slotted-Aloha type MAC protocol with selfish behaviors. We propose a generalized Markov model for slotted-Aloha protocol. Under this model, we don't assume that each node knows the perfect information, which is the current number of backlogged nodes in the system. We analyze the performance of the protocol when nodes are either cooperative or non-cooperative. The main results are: (1) Under cooperative selfish behaviors, one half of the medium utilization is achievable. (2) Under non-cooperative selfish behaviors, medium utilization collapses and Prisoner's Dilemma characterizes the situation.

2. MARKOV MODEL

In this section, we build a Markov Model for slotted-Aloha type MAC protocol. Recall the slotted-Aloha protocol:

1. If a node has a new packet to send, it sends at the beginning of the next time-slot.
2. If a node successfully transmitted its packet, it can transmit a new packet in the next time-slot.
3. If a node detects a collision, it retransmits its packet in each subsequent time-slot with probability p until the packet is successfully transmitted.

In a slotted-Aloha protocol, each node is in either a backlogged state or a free state. The transmission decision only depends on the state of the node. Therefore, the decision in slotted-Aloha for each node is actually Markovian. Suppose nodes in the system always have packets to transmit, we have the following notations for a generalized Markov model for slotted-Aloha type MAC protocols.

$N =$ Number of nodes in the system.
$p_1^x =$ Transmitting probability at free states for node x.
$p_2^x =$ Transmitting probability at backlogged states for node x.
$Th_x =$ Throughput function, which indicates the average throughput of node x.
$C_x =$ Cost function, which indicates the average transmitting probability of node x in each time-slot. If transmitting a packet in a time-slot incurs a unit cost (e.g. power consumption), C_x represents the average cost for the node.

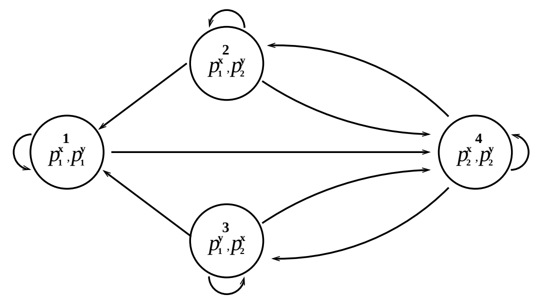

Figure 1: Two-node Markov Chain.

Figure 1 shows the Markov Chain for two nodes. It's easy to extend this Markov Chain for N nodes where the chain would be consist of 2^N states. The transition matrix for the above Markov Chain is:

$$P = \begin{pmatrix} 1-p_1^x p_1^y & 0 & 0 & p_1^x p_1^y; \\ (1-p_1^x)p_2^y & 1-p_2^y & 0 & p_1^x p_2^y; \\ (1-p_1^y)p_2^x & 0 & 1-p_2^x & p_1^y p_2^x; \\ 0 & p_2^x(1-p_2^y) & p_2^y(1-p_2^x) & p_{44} \end{pmatrix}$$

where $p_{44} = p_2^x p_2^y + (1-p_2^x)(1-p_2^y)$.

If $p_1^x, p_1^y, p_2^x, p_2^y > 0$, the Markov Chain is positive-recurrent. The steady state distribution is the following:

$$\vec{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{pmatrix} = \frac{1}{k_1+k_2+k_3+k_4} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}$$

where

$$\vec{k} = \begin{pmatrix} p_2^x p_2^y[(1-p_1^x)p_2^x(1-p_2^y)+(1-p_2^x)p_2^y(1-p_1^y)] \\ p_1^x p_1^y (p_2^x)^2(1-p_2^y) \\ p_1^x p_1^y (p_2^y)^2(1-p_2^x) \\ p_1^x p_1^y p_2^x p_2^y \end{pmatrix}$$

The corresponding throughput and cost functions of node x are:

$$\begin{aligned} Th_x &= \pi_1(p_1^x)(1-p_1^y) + \pi_2(p_1^x)(1-p_2^y) + \\ &\quad \pi_3(p_2^x)(1-p_1^y) + \pi_4(p_2^x)(1-p_2^y). \\ C_x &= \pi_1(p_1^x) + \pi_2(p_1^x) + \pi_3(p_2^x) + \pi_4(p_2^x). \end{aligned}$$

3. COOPERATIVE PERFORMANCE ANALYSIS

In this section, we assume each node behaves selfishly but cooperatively. It means that each node tries to help the system get better performance so that they individually gain better performance. We try to answer the following questions.

1. What are the best value of p_1^x and p_2^x for each node x in order to achieve the best performance for the system.

2. What is the achievable performance lower bound for the system?

3. How fair is the protocol treating individual nodes?

If the protocol is biased to one of the nodes, it can always fully utilized the system by allowing only one node to transmit at all time. If we had centralized scheduler, we could also make fair share of the medium with 100% utilization. Here, we seek unbiased and distributed solution for all nodes, so that they have the same performance on average.

Theorem 1. *For two homogeneous nodes with* $p_1^x = p_1^y = p_1$ *and* $p_2^x = p_2^y = p_2$, $\sup\{Th_x + Th_y\} = 2/3$.

Proof: Substitute with p_1 and p_2, we have

$$\vec{k} = \begin{pmatrix} 2p_2(1-p_2)(1-p_1) \\ (1-p_2)p_1^2 \\ (1-p_2)p_1^2 \\ p_1^2 \end{pmatrix}$$

$$\begin{aligned} Th_x &= \pi_1 p_1(1-p_1) + \pi_2 p_1(1-p_2) + \\ &\quad \pi_3 p_2(1-p_1) + \pi_4 p_2(1-p_2) \\ &= \beta p_1(p_1^2 - \alpha p_1 + \alpha)/(p_1^2 - \alpha\beta p_1 + \alpha\beta) \end{aligned}$$

where

$$\alpha = 2p_2,\ \beta = (1-p_2)/(3-2p_2).$$

When $p_1 = 1$, $Th_x = \beta = (1-p_2)/(3-2p_2)$ and $\beta \to 1/3$ as $p_2 \to 0$. By symmetry, $Th_x + Th_y \to 2/3$ as $p_2 \to 0$.

Next, we want to show $Th_x < 1/3$ for all $p_1, p_2 \in (0, 1]$. It's equivalent to show the following:

$$\begin{aligned} & \beta p_1(p_1^2 - \alpha p_1 + \alpha)/(p_1^2 - \alpha\beta p_1 + \alpha\beta) < 1/3 \\ \iff & 3\beta p_1(p1^2 - \alpha p_1 + \alpha) < p_1^2 - \alpha\beta p_1 + \alpha\beta \\ \iff & 3\beta p_1^3 - (3\alpha\beta + 1)p_1^2 + 4\alpha\beta p_1 - \alpha\beta < 0 \end{aligned}$$

Let's define $f(p_1) = 3\beta p_1^3 - (3\alpha\beta+1)p_1^2 + 4\alpha\beta p_1 - \alpha\beta$. $f(0) = -\alpha\beta < 0$ and $f(1) = 3\beta - 1 < 0$. Since $f(p_1)$ is a cubic function of p_1, it's sufficient to show that the local maximum is less than zero, in order to prove that for any $p_1 \in (0, 1]$, $f(p_1) < 0$. At the local maximum,

$$f^{'}(p_1) = 9\beta p_1^2 - 2(3\alpha\beta + 1)p_1 + 4\alpha\beta = 0.$$

Using the above condition, it's equivalent to show

$$-(1/3)(3\alpha\beta + 1)p_1^2 + (8/3)\alpha\beta p_1 - \alpha\beta < 0.$$

The maximum of the above function is

$$[(4/3)(3\alpha\beta + 1)\alpha\beta - ((8/3)\alpha\beta)^2]/[-(4/3)(3\alpha\beta + 1)].$$

Therefore, it's sufficient to show

$$\begin{aligned} & (4/3)(3\alpha\beta + 1)\alpha\beta - ((8/3)\alpha\beta)^2 > 0 \\ \iff & (4/3)(3\alpha\beta + 1) - (64/9)\alpha\beta > 0 \\ \iff & \alpha\beta < 3/7 \\ \iff & 2p_2(1-p_2)/(3-2p_2) < 3/7 \\ \iff & 14p_2^2 - 20p_2 + 9 > 0 \end{aligned}$$

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The solution $\{p_1 = 1,\ p_2 \to 0\}$ is consistent with our intuition: when the medium is not busy, we use it; when it becomes busy, we stop using it. Theorem 1 tells us that slotted-Aloha should use small transmitting probabilities during the backlogged state to get the best performance for two nodes.

Theorem 2. *For* N *homogeneous nodes with* $p_1 = 1$, $p_2 \to 0$, *total throughput tends to* $\frac{N}{2N-1}$.

Proof: We classify the system states as the following two. One state is the busy state when only one of the nodes is transmitting. The other state is either when the system is idle or when collisions happen.

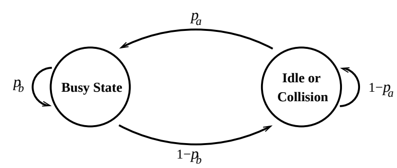

Figure 2: N-node Markov Chain with $\{p_1 = 1, p_2 \to 0\}$.

The transition probabilities are: $p_b = (1-p_2)^{N-1}$ and $p_a = Np_2(1-p_2)^{N-1}$. p_b indicates the probability that all of the $N-1$ backlogged nodes don't transmit. p_a indicates the probability that only one of the N nodes transmits. The system utilization becomes:

$$\begin{aligned} \rho &= \pi_{busy} = p_a/(1 - p_b + p_a) \\ &= \frac{Np_2(1-p_2)^{N-1}}{1-(1-p_2)^{N-1} + Np_2(1-p_2)^{N-1}} \\ &= N/(\frac{1}{p_2}\frac{1-(1-p_2)^{N-1}}{(1-p_2)^{N-1}} + N) \\ &= N/(\frac{1+(1-p_2)+(1-p_2)^2+\cdots+(1-p_2)^{N-2}}{(1-p_2)^{N-1}} + N) \end{aligned}$$

Therefore, $\rho \to \frac{N}{2N-1}$ as $p_2 \to 0$. ■

By Theorem 1, we can achieve a total throughput as close as $2/3$ for two nodes. But when $p_2 \to 0$, nodes need to backoff for a very long time, causing short term unfairness.

In general, after one node successfully transmitted a packet, it obtains the medium. This node will continue to occupy the medium for a random amount of time, say X_b. X_b is a binomial random variable with parameter $1 - p_b$. When X_b is smaller, short term fairness of the system improves.

Suppose we want to achieve $E[X_b] = M$ for some constant M, then $M = \frac{1}{1-(1-p_2)^{N-1}}$. Therefore, the total throughput becomes a function of M:

$$\rho = \frac{Np_2(1-p_2)^{N-1}}{1 + (Np_2 - 1)(1-p_2)^{N-1}} = \frac{N(M-1)}{N(M-1) + \frac{1}{1-\sqrt[N-1]{1-\frac{1}{M}}}}$$

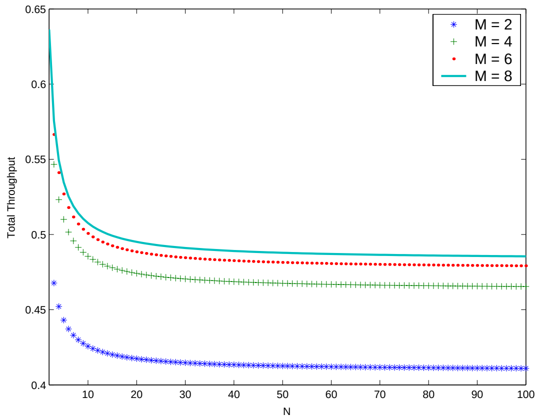

Figure 3: Throughput under different fairness conditions.

Figure 3 shows the total throughput under different values of M and N. If we can bear with $M = 8$, we can achieve a total throughput close to 0.5 even for large N (When $N \to \infty$, the total throughput doesn't collapse to zero.).

Theorem 3. *Under any fairness condition* $E[X_b] = M$, $\rho \to 1/[1 - (M-1)\ln(1-\frac{1}{M})]$ *as* $N \to \infty$.

Proof:

$$\frac{1}{M-1}\frac{1}{\frac{1}{\rho}-1} = (1 - \sqrt[N-1]{1-\frac{1}{M}})N = \frac{1 - \sqrt[N-1]{1-\frac{1}{M}}}{\frac{1}{N}}$$

Right hand side is in the form of $\frac{0}{0}$ as $N \to \infty$. By L'hospital rule,

$$\lim_{N\to\infty} \frac{\ln(1-\frac{1}{M})\sqrt[N-1]{1-\frac{1}{M}}(N-1)^{-2}}{-N^{-2}} = -\ln(1-\frac{1}{M})$$

$$\therefore \lim_{N\to\infty} \frac{1}{M-1}\frac{1}{\frac{1}{\rho}-1} = -\ln(1-\frac{1}{M})$$

$$\Rightarrow \lim_{N\to\infty} \rho = \frac{1}{1-(M-1)\ln(1-\frac{1}{M})}$$

■

4. NON-COOPERATIVE GAME FRAMEWORK AND PRISONER'S DILEMMA

In last section, we obtained the best solutions for all nodes to achieve high system performance. In this section, we assume that each node is only interested in its own performance, and wants to maximize its own throughput. We first formulate this problem as a constrained optimization problem for each node. After solving a specific "leader-follower" game, we identify that Prisoner's Dilemma phenomena happens when nodes are allowed to transmit most of the time.

4.1 Two-node Leader-follower Game

We consider two nodes x and y which have their own budget constraints: $C_x \leq B_x$ and $C_y \leq B_y$. $B_x, B_y \in (0,1]$ are two budget constants which physically restricts the average number of packets (or the battery power consumption) the node can transmit in each time-slot. The non-cooperative leader-follower game can be described formally as follows:

Player:	The leader node x and the follower node y.
Strategy:	$S^x = \{p_1^x, p_2^x\}$ for x; $S^y = \{p_1^y, p_2^y\}$ for y.
Payoff:	Th_x and Th_y for x and y respectively.
Game rule:	x decides $\{p_1^x, p_2^x\}$ first.
	y decides $\{p_1^y, p_2^y\}$ after knowing $\{p_1^x, p_2^x\}$.

For any given $\widetilde{S^x}$, the follower node y solves:

$$\hat{S^y}(\widetilde{S^x}) = arg\ max\ Th_y(\widetilde{S^x}, \hat{S^y})$$
$$\texttt{Subject to}: C^y(\widetilde{S^x}, \hat{S^y}) \leq B_y.$$

The leader node x solves:

$$\hat{S^x} = arg\ max\ Th_x(\hat{S^x}, \hat{S^y}(\hat{S^x}))$$
$$\texttt{Subject to}: C_x(\hat{S^x}, \hat{S^y}(\hat{S^x})) \leq B_x.$$

4.2 Three Solution regimes

We solve the above leader-follower game for nodes who have the same budget constraints, which is $B_x = B_y$.

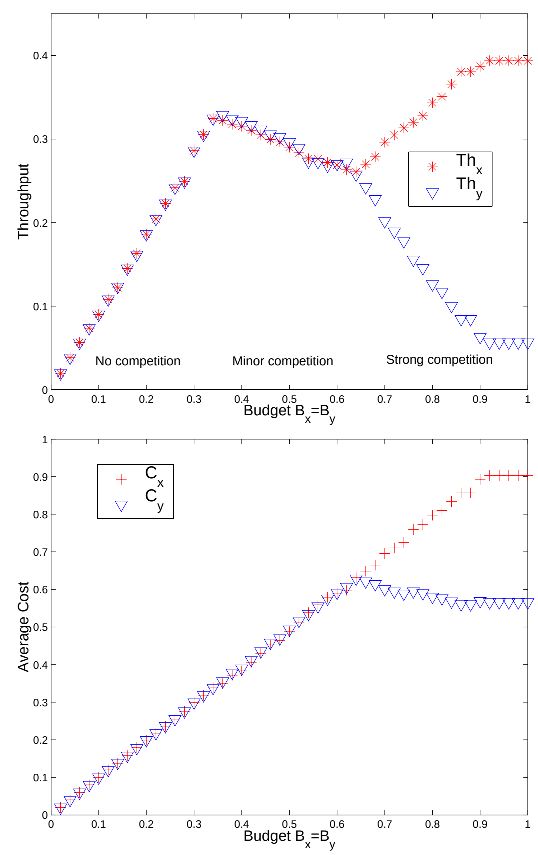

Figure 4: Throughput and cost in the leader-follower game.

Figure 4 shows the throughput and costs of both players. We identify three regimes in the game solutions:

1. When the budget is less than $1/3$, both players gain the same throughput. They use up their budgets. The throughput is mainly limited by the budget constraints, but not the competition between these two players.
2. When the budget is between $1/3$ and $2/3$, both players gain similar throughput. But the throughput keeps decreasing when the budget increases. The throughput is limited by both the budget constraints and the competition between these two players.
3. When the budget is more than $2/3$, leader takes advantage of the follower and gains much more throughput. The competition becomes strong. But both players don't use up their budgets.

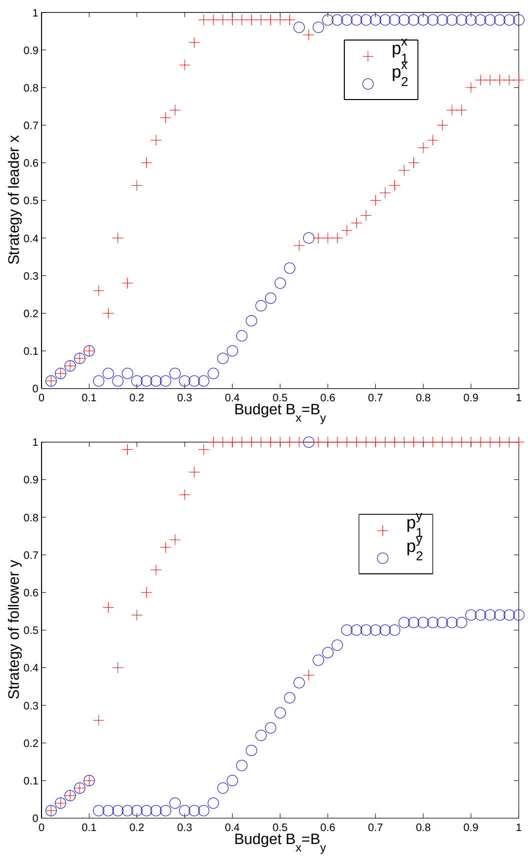

Figure 5: Strategies in the leader-follower game.

Figure 5 shows the strategy solutions of both players. In the first two solution regimes, both players use similar strategies. Especially, when the budget is close to $1/3$, both players use the best cooperative strategy and achieve a total throughput close to $2/3$. After that, because the budget is higher, they can afford to use more aggressive and selfish strategy to increase p_2. When the budget is more than $2/3$, the leader's strategy becomes totally different. It keeps transmitting during the backlogged state, forcing the follower to backoff.

4.3 Prisoner's Dilemma Phenomena

We pick up one typical strategy solution in each of the solution regimes:

- No competition regime: $B_x = B_y = 0.34$. Strategy $S^x = S^y = S_C = \{p_1 = 1, p_2 = 0.02\}$.
- Minor competition regime: $B_x = B_y = 0.5$. Strategy $S^x = S^y = S_M = \{p_1 = 1, p_2 = 0.28\}$.
- Strong competition regime: $B_x = B_y = 0.8$. Strategy $S^x = S_L = \{p_1 = 0.64, p_2 = 0.98\}$, $S^y = S_F = \{p_1 = 1, p_2 = 0.52\}$.

Now, consider the following two simultaneous move game:

	S_C	S_M
S_C	(0.3311,0.3311)	(0.0022,0.9413)
S_M	(0.9413,0.0022)	**(0.2845,0.2845)**

In the first simultaneous move game, the most efficient solution is at (S_C, S_C). But the symmetric Nash Equilibrium is at (S_M, S_M) and the payoff in equilibrium is worse than that in the cooperative solution. Here, we see a typical Prisoner's Dilemma situation. Although both players know it's the best for them to be at (S_C, S_C), strategy S_M should always be played. Because S_C is strictly dominated by S_M, which means regardless of the strategy of the opponent, S_M is always better than S_C for the player to use. This is also the reason why the leader cannot take advantage of the follower in the second solution regime.

	S_F	S_L
S_F	(0.2449,0.2449)	**(0.1263,0.3433)**
S_L	**(0.3433,0.1263)**	(0.0324,0.0324)

In the second simultaneous move game, each player choose either the leader's strategy or the follower's strategy. The best solution is at (S_F, S_F). But this solution is not an equilibrium. Each player has an incentive to move from S_F to S_L to get higher throughput. Different from the previous game, when both players use S_L, the total throughput goes close to zero. In this game, we have two asymmetric Nash equilibra instead of one symmetric one. This is also the reason why the leader can take advantage of the follower in the third solution regime.

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