Generalized Slotted-Aloha in Cooperative, Competitive and Adversarial Environments

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***Abstract*— Aloha [1] and its slotted variation [2] are commonly deployed Medium Access Control (MAC) protocols in environments where multiple transmitting devices compete for a medium, yet may have difficulty sensing each other's presence. This is also known as the "hidden terminal problem". Competing 802.11 [3] gateways, as well as most modern digital cellular systems, like GSM[4], are examples. This paper models and evaluates the throughput that can be achieved in a system where nodes compete for bandwidth using a generalized version of slotted-Aloha protocols. The protocol is implemented as a two-state system, where the probability that a node transmits in a given slot depends on whether the node's prior transmission attempt was successful. Using Markov Models, we evaluate the channel utilization and fairness of these types of protocols for a variety of node objectives, including maximizing aggregate throughput of the channel, each node greedily maximizing its own throughput, and attacker nodes that attempt to jam the channel. If all nodes are selfish and greedily attempt to maximize their own throughputs, a situation similar to the traditional Prisoner's Dilemma[5] arises. Our results reveal that under heavy loads, a greedy strategy reduces the utilization, and that attackers cannot do much better than attacking during randomly selected slots.**

I. Introduction

In many communication networks, the communication medium is often shared by multiple users who must compete for access. In Ethernet[6], nodes use CSMA/CD [7], [8] as a MAC protocol. In order to reduce the probability of collisions, each node implements CSMA/CD, sensing the medium to ensure its availability prior to transmitting. However, for wireless ad-hoc networks or sensor networks, carrier sensing may not be effective. This is because nodes may not be able to sense one anothers' presence, yet their transmissions may still interfere. Ad hoc networks, sensor networks, and competing "hotspot" 802.11 gateways are examples where this so-called "hidden terminal problem" occurs.

The Aloha protocol [1] is a fully decentralized medium access control protocol that does not perform carrier sensing. The subsequent slotted-Aloha[2] protocol was introduced to improve the utilization of the shared medium by synchronizing the transmission of devices within time-slots. Today, various forms of slotted-Aloha protocols are widely used in most of the current digital cellular networks, such as the Global System for Mobile communications (GSM)[1].

Because slotted-Aloha exhibits an instability as the number of transmitting nodes increases [9], [10], early research focused on stability control [11], [12]. However, today's networks often implement an admission control procedure to limit the number of simultaneous users in the system at any time. In this sense, the system itself is stable in terms of users.

In this work, we consider a generalization of the slotted-Aloha protocol. Like slotted-Aloha, the decision to transmit within a slot has a random component. However, in traditional slotted-Aloha, the user continues transmission in subsequent slots until a subsequent collision. In our generalized version, the user may cease transmitting with some fixed (non-zero) probability. We model a system of N users implementing this generalized protocol with tunable parameters via Markov Models that allow us to measure the rate at which nodes attempt to transmit packets (cost), and their rates of success (throughput). In parts, we impose budget constraints that restrict the nodes' costs, such that the fraction of slots within which a node attempts transmissions is bounded. In practice, this may be due to energy constraints, or a bandwidth constraint placed on the network application.

This generalized version of slotted-Aloha is worth studying for two reasons. First, it is derived from a protocol that is commonly used today. Second, we will show that the generalized versions can outperform the original version, both in terms of aggregate throughput, as well as the ability to cope with malicious users.

We begin by exploring an environment where N users cooperate and set the protocol parameters to maximize the total system throughput while sharing the bandwidth evenly. We find that the throughput is bounded by $N/(2N-1)$ and that to achieve this utilization, users who gain access to the channel must transmit over a large number of consecutive slots. We then explore how throughput decreases as "short-term fairness" is more strictly enforced, reducing the expected number of consecutive slots.

Next, we consider greedy users who wish to maximize their own throughputs, perhaps at the expense of the nodes against whom they compete. We evaluate this setting as a game, where nodes set their parameters in a greedy manner and other nodes subsequently modify their parameters in response to maximize their own throughputs. We find that performance of the protocol is a function of the nodes' budgets, and takes on three distinct behaviors. When nodes' budgets are low, the greedy strategy is optimal. When nodes' budgets fall within a medium range, there is a unique equilibrium point where all nodes achieve the same throughput, but these throughputs are less than what would be obtained in a cooperative setting. When nodes' budgets fall within the highest range, then there

[1]In the GSM network, the control channels of the TDM channels use slotted-Aloha.

are multiple equilibrium points, and the throughput achieved by a node depends on when it took its turn in the game. We develop an additional enhancement to the protocol that can be implemented by cooperative nodes which will encourage greedy users to tune their protocol parameters to match those of a cooperative node, maximizing throughput.

Last, we consider an attacking node that, with a limited budget, seeks to minimize the throughput of the other nodes in the system. We show that when the attacker's budget is small, selecting random slots (i.e., via a Bernoulli process) is optimal. When the budget is large, the optimal strategy is to mimic a greedy user. Our analysis provides insights on the limits of success a jammer can have in disrupting a slotted-Aloha like network.

The main contribution of this paper can be summarized as follows:

1) We formulate different user behaviors under a generalized slotted-Aloha protocol where users make decisions using a two-state system.
2) We identify throughput bounds for a system of cooperative users and explore the trade-off between user throughput and short-term fairness.
3) Under non-cooperative/selfish behaviors of the users, we identify a Prisoner's Dilemma phenomenon and propose methods to detect and prevent nodes from acting selfishly without regard for other nodes' throughputs.
4) Under adversarial behavior of one user, we measure the maximum possible deterioration of the system and try to understand the behavior of an attacker.

We organize our paper as follows. In Section II, we review related work. In Section III, we motivate the protocol and construct a Markov Model for the generalized slotted-Aloha protocol. In Section IV, we measure the system throughput in a cooperative environment where users want to maximize the total throughput of the system. In Section V and VI, we evaluate both the aggregate and individual user throughputs where selfish users exist in the system. We formulate the game as a Prisoner's Dilemma situation in Section V and in Section VI present strategies that cooperative nodes can implement to detect and prevent selfish user behaviors. In section VII, we explore a system in which attackers try to minimize the throughputs of the remaining nodes. Section VIII concludes.

II. Related Work

The Aloha protocol and its slotted version have been studied for decades from the early seventies. Due to Aloha's instability [13], [9], [14], [15], [11] in nature, early research focused on stabilizing the Aloha protocols[11], [12]. Rivest[12] proposed a pseudo-Bayesian algorithm to stabilize Aloha that utilized feedback to estimate the number of current backlogged nodes in the system.Later, even many performance evaluations of the Aloha protocols were accompanied with dynamic controls[16], [10] to stabilize the systems.

In this work, we focus on the performance of stable slotted-Aloha type systems, where only a finite number of users will access the shared medium simultaneously. The justification of this assumption relies on the implementation of admission control procedures in today's networks. Early work on slotted Aloha with finite number of users can be found in [13].

Besides finding the throughput bounds for a finite slotted-Aloha type system, we consider the performance of individual users under different user behaviors. We find that users always have incentive not to follow the designed protocol (not backoff) in order to achieve higher throughput. Thus, Game Theory[5], [17] applies here.

Recent work using Game Theory to analyze users behaviors in MAC protocols and wireless ad-hoc networks can be found in [18], [19] and [20], [21] respectively. More specifically, game-theoretical analysis of the Aloha protocols can be found in [22][2],[23], [24], [25], [26].

MakKenzie and Wicker's work [23], [24] discussed the stability of slotted-Aloha with selfish user behaviors and perfect information. Our work is different in three ways. First, we focus on performance (attainable throughput) instead of stability. In terms of data backlog at the users, we consider scenarios of *elastic transfers*, where users always have data to send and utilize whatever bandwidth is available, and hence classical stability results do not apply to our analysis. Second, we assume that nodes do not know the number of nodes that attempt to transmit in a particular slot, and know only whether or not their transmission succeeded or failed after the fact. Third, we consider cooperative and attacking strategies in addition to greedy strategies.

Jin and Kesidis's work [25] discussed the equilibra of a noncooperative game for Aloha protocols. In their noncooperative game formulation, each user only uses one transmitting probability (i.e., always in a backlogged state). Moreover, utility functions and payments are specified for each user. In our work, on the other hand, the formulation is for a generalized slotted-Aloha protocol which considers the Markovian decisions depending on whether the most recent transmission is a success (in a Free State) or a failure (in a Backlogged State). And we do not impose any payment on the users. Our settings capture more realistic features in real Aloha systems.

Altman et al. [26] consider slotted-Aloha systems as both cooperative and noncooperative games with partial information. Their work assumes that there are a finite number of sources without buffer. The arrival packets to each source follows a Bernoulli process. As in typical slotted-Aloha, users only control the backlog probability in both games. In our work, we consider the saturated arrival when each user always has packets to transmit. But users' strategies are more broad. Because users are also allowed to choose a non-zero probability to backoff even its previous transmission is a success. In addition, we analyze an adversarial game where an attacker who wants to minimize other users' throughput appears in the game.

III. Protocol Description and Model

In this section, we describe a generalized slotted-Aloha MAC protocol and construct a Markov Model from which its throughput can be measured. First, let us overview the original slotted-Aloha protocol:

[2]It is a preliminary abstract of this paper.

1) Time is divided into slots, and each node can attempt to send one packet in a slot.
2) If a node has a new packet to send, it attempts transmission during the next time-slot.
3) If a node successfully transmitted its packet, it can transmit a new packet in the next time-slot.
4) If a node detects a collision, it retransmits its packet in each subsequent time-slot with probability p until the packet is successfully transmitted. Returning to step 3 after a successful transmission.

The slotted-Aloha protocol described above can be implemented as a 2-state system, where the state maintains the outcome of the previously attempted transmission. A node is in its *Free State* if the most recent transmission from that node is a success. Otherwise, the node is in its *Backlogged State*. In the Free State, a node transmits during the next slot with probability 1, and in the Backlogged State, it transmits during the next slot with probability p. Our generalization of the above protocol is to allow a node to vary the probability with which a node transmits a packet when it resides within the Free State.

Our evaluation will consider a network of N nodes, where often N will start as 2. We assume that nodes are able to coordinate slot transmission times and can estimate the number of nodes N with which they compete for bandwidth. However, because nodes' transmissions may interfere but cannot be deciphered, methods to prevent slot contention that require explicit communication and coordination among the competing members (e.g., TDMA, RTS-CTS) cannot be used.

Each node x can tune its protocol using two parameters:

$p_1^x =$ Transmitting probability in the Free State for node x.

$p_2^x =$ Transmitting probability in the Backlogged State for node x.

Given N and the transmitting probabilities for each of the nodes in each of the states, it is possible to compute the following measures of system performance:

$T_x =$ The *throughput* of node x, which is the fraction of slots within which x successfully completes a transmission, in that it is the only device to attempt transmission within a slot.

$C_x =$ The *cost* for node x, which is the fraction of slots within which x attempts transmission (regardless of whether the transmission fails or succeeds).

Nodes may have physical limitations (e.g. power consumption constraints or application throughput constraints) that may bound its cost function. We bound allowed cost by a *budget*, B_x, such that a node's parameters must produce a cost $C_x \leq B_x$.

When we consider cooperating nodes that seek to maximize throughput, we are also interested in system *fairness*: all nodes should get an equal share of the throughput. In addition, we assume that it is undesirable for any one node to "capture" the medium for an extended number of slots - a long-term capture can be thought of as unfair over a short duration. Koksal's work[27] gives an analysis of the short-term fairness of MAC protocols. It provides some insight into why MAC protocols exhibit bad short-term fairness using two different fairness indexes. In this paper, we measure short-term fairness via a more fundamental quantity defined as the following:

Definition *1:* Let D_x be the number of consecutive slots following an initially successful transmission over which node x successfully transmits packets (i.e., if there are k successful consecutive transmissions, then $D_x = k - 1$). The system is said to be M-short-term fair to all nodes if $E[D_x] \leq M$ for all nodes x.

Each node's decision to transmit within a particular slot depends only on the outcome of its previous attempt (success or failure), and does not depend on the state of other nodes. Hence, this protocol is easily implemented in a distributed manner. Moreover, each node's decision is in fact Markovian, as it depends only on the previous attempt's outcome. For simplicity, we will assume that a node always has a packet to send in a slot whenever our slotted-Aloha variant decides to transmit a packet in a slot (i.e., nodes have a sufficient backlog of packets). However, our model also easily captures the case where a packet enters the queue to be transmitted with a fixed probability.[3]

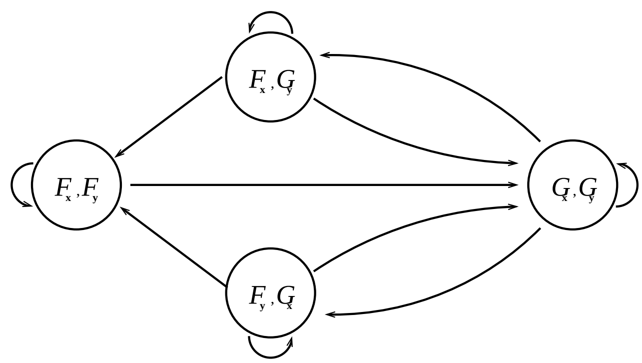

Fig. 1. Two-node Markov Chain.

Figure 1 shows the state transition diagram for a two-node system with node x and y. F_x and G_x represent that node x is in a free state and a backlogged state respectively. A system for N nodes is easily modeled by as a Markov Model where the chain would consist of 2^N states. By numbering the states $(F_x, F_y), (F_x, G_y), (F_y, G_x), (G_x, G_y)$ to be $1, 2, 3, 4$, the transition matrix for a two-node Markov Model is:

$$P = \begin{pmatrix} 1 - p_1^x p_1^y & 0 & 0 & p_1^x p_1^y; \\ (1 - p_1^x) p_2^y & 1 - p_2^y & 0 & p_1^x p_2^y; \\ (1 - p_1^y) p_2^x & 0 & 1 - p_2^x & p_1^y p_2^x; \\ 0 & p_2^x (1 - p_2^y) & p_2^y (1 - p_2^x) & p_{44} \end{pmatrix}$$

where $p_{44} = p_2^x p_2^y + (1 - p_2^x)(1 - p_2^y)$.

If $p_1^x, p_1^y, p_2^x, p_2^y > 0$, the Markov Model is positive-recurrent. The steady state distribution is the following:

$$\vec{\pi} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{pmatrix} = \frac{1}{k_1 + k_2 + k_3 + k_4} \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}$$

[3]The model simply has to reduce the probability of transmitting in the free state to also account for the steady-state probability that a new packet arrived during the previous attempt to transmit a packet.

where

$$\vec{k} = \begin{pmatrix} p_2^x p_2^y[(1-p_1^x)p_2^x(1-p_2^y)+(1-p_2^x)p_2^y(1-p_1^y)] \\ p_1^x p_1^y (p_2^y)^2(1-p_2^y) \\ p_1^x p_1^y (p_2^y)^2(1-p_2^x) \\ p_1^x p_1^y p_2^x p_2^y \end{pmatrix} \quad (1)$$

The corresponding throughput and cost functions of node x are:

$$\begin{aligned} T_x &= \pi_1(p_1^x)(1-p_1^y)+\pi_2(p_1^x)(1-p_2^y)+ \\ &\quad \pi_3(p_2^x)(1-p_1^y)+\pi_4(p_2^x)(1-p_2^y). \end{aligned} \quad (2)$$

$$C_x = \pi_1(p_1^x)+\pi_2(p_1^x)+\pi_3(p_2^x)+\pi_4(p_2^x). \quad (3)$$

IV. **Cooperative Performance Analysis**

In this section, we assume that nodes cooperate to fairly (i.e., equally) share the available bandwidth to maximize the aggregate system throughput. By doing so, each node achieves the maximum throughput possible in a fair allocation when limited to protocols that cannot sense the line. Clearly, if it were permissible to bias the allocation toward one of the nodes, the system could achieve full utilization by allowing only one node to transmit at all time. If a centralized scheduler or carrier sensing mechanism were permitted, we could also make fair share of the medium with 100% utilization. Here, we seek an unbiased and distributed solution for all nodes such that nodes will achieve the same performance on average.

Our goal in this section is to answer the following questions:

1) What values of p_1^x and p_2^x for each node x to maximize the total throughput of the system?
2) What is the maximum achievable throughput of the system?
3) What is the short-term fairness of the optimal allocation, and how can that short-term fairness be improved?

Theorem *1:* For two homogeneous nodes with $p_1^x = p_1^y = p_1$ and $p_2^x = p_2^y = p_2$, $\sup\{T_x + T_y\} = 2/3$.

Proof: Substitute all the transmitting probabilities with p_1 and p_2 into Equation (1), we have

$$\vec{k} = \begin{pmatrix} 2p_2(1-p_2)(1-p_1) \\ (1-p_2)p_1^2 \\ (1-p_2)p_1^2 \\ p_1^2 \end{pmatrix}$$

$$\begin{aligned} T_x &= \pi_1 p_1(1-p_1)+\pi_2 p_1(1-p_2)+ \\ &\quad \pi_3 p_2(1-p_1)+\pi_4 p_2(1-p_2) \\ &= \beta p_1(p_1^2-\alpha p_1+\alpha)/(p_1^2-\alpha\beta p_1+\alpha\beta) \end{aligned}$$

where

$$\alpha = 2p_2,\ \beta = (1-p_2)/(3-2p_2).$$

When $p_1 = 1$, $T_x = \beta = (1-p_2)/(3-2p_2)$ and $\beta \to 1/3$ as $p_2 \to 0$. By symmetry, $T_x + T_y \to 2/3$ as $p_2 \to 0$.

Next, we want to show $T_x < 1/3$ for all $p_1, p_2 \in (0,1]$. It is equivalent to show the following:

$$\begin{aligned} & \beta p_1(p_1^2-\alpha p_1+\alpha)/(p_1^2-\alpha\beta p_1+\alpha\beta) < 1/3 \\ \Longleftrightarrow\ & 3\beta p_1(p1^2-\alpha p_1+\alpha) < p_1^2-\alpha\beta p_1+\alpha\beta \\ \Longleftrightarrow\ & 3\beta p_1^3-(3\alpha\beta+1)p_1^2+4\alpha\beta p_1-\alpha\beta < 0 \end{aligned}$$

Let us define $f(p_1) = 3\beta p_1^3-(3\alpha\beta+1)p_1^2+4\alpha\beta p_1-\alpha\beta$. Two boundary conditions are $f(0) = -\alpha\beta < 0$ and $f(1) = 3\beta-1 < 0$. Since $f(p_1)$ is a cubic function of p_1, it is sufficient to show that the local maximum is less than zero, so as to prove that for any $p_1 \in (0,1]$, $f(p_1) < 0$. At the local maximum,

$$f'(p_1^*) = 9\beta(p_1^*)^2-2(3\alpha\beta+1)p_1^*+4\alpha\beta = 0.$$

Using the above condition, it is equivalent to show

$$f(p_1^*) = -(1/3)(3\alpha\beta+1)(p_1^*)^2+(8/3)\alpha\beta p_1^*-\alpha\beta < 0.$$

The maximum of the above function is

$$[(4/3)(3\alpha\beta+1)\alpha\beta-((8/3)\alpha\beta)^2]/[-(4/3)(3\alpha\beta+1)].$$

The denominator is negative, while the numerator is positive because:

$$\begin{aligned} & (4/3)(3\alpha\beta+1)\alpha\beta-((8/3)\alpha\beta)^2 > 0 \\ \Longleftrightarrow\ & (4/3)(3\alpha\beta+1)-(64/9)\alpha\beta > 0 \\ \Longleftrightarrow\ & \alpha\beta < 3/7 \\ \Longleftrightarrow\ & 2p_2(1-p_2)/(3-2p_2) < 3/7 \\ \Longleftrightarrow\ & 14p_2^2-20p_2+9 > 0 \end{aligned}$$

Finally, because the local maximum $f(p_1^*) < 0$, we conclude that $f(p_1) < 0$ for all $p_1 \in [0,1)$. ∎

Theorem 1 upper-bounds the maximum fair throughput at $2/3$, which is achieved in the limit as both nodes choose $\{p_1 = 1, p_2 \to 0\}$. This solution is intuitive: collisions are less likely to occur in an carrier-sense free environment when nodes are very unlikely to start trying to transmit, but hold the medium until a subsequent collision.

Theorem *2:* For N homogeneous nodes with $p_1 = 1$, $p_2 \to 0$, the total throughput tends to $\frac{N}{2N-1}$.

Proof: Consider in each time-slot, the whole system is in certain state. We aggregate all the system states into the following two states. One state is the "Busy" state where only one of the nodes is transmitting in the time-slot. The other state is the "Idle or Collision" state where no node or more than one node are transmitting in the time-slot. The state transition diagram is shown in Figure 2.

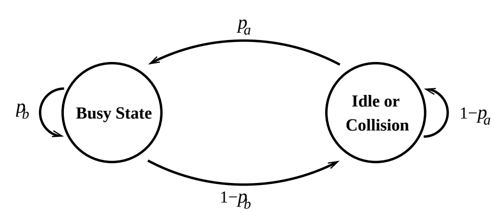

Fig. 2. N-node Markov Chain with $\{p_1 = 1, p_2 \to 0\}$.

We define the transition probabilities to be: $p_b = (1-p_2)^{N-1}$ and $p_a = Np_2(1-p_2)^{N-1}$. p_b indicates the probability that all of the $N-1$ backlogged nodes do not transmit. p_a indicates the probability that only one of the N nodes transmits. The system utilization becomes:

$$\begin{aligned} \rho &= \pi_{busy} = p_a/(1-p_b+p_a) \\ &= \frac{Np_2(1-p_2)^{N-1}}{1-(1-p_2)^{N-1}+Np_2(1-p_2)^{N-1}} \\ &= N/(\frac{1}{p_2}\frac{1-(1-p_2)^{N-1}}{(1-p_2)^{N-1}}+N) \\ &= N/(\frac{1+(1-p_2)+(1-p_2)^2+\cdots+(1-p_2)^{N-2}}{(1-p_2)^{N-1}}+N) \end{aligned}$$

Therefore, $\rho \to \frac{N}{2N-1}$ as $p_2 \to 0$. ∎

Intuitively, when the number of nodes increases in the system, the system throughput decreases. However, Theorem 3 shows that the throughput does not drop to zero: even when the number of nodes tends to infinity, we can still achieve a total throughput of one half. Note that this result differs from the traditional performance bound ($1/e$) of slotted-Aloha because our generalized model permits "capture" of the resource, allowing it to be used while all other nodes are back off. A different analysis of the "Capture Phenomenon" appears in [13].

Although the solution $\{p_1 = 1, p_2 \to 0\}$ maximizes throughput, it is not short-term fair. As $p_2 \to 0$, we have $E[D_x] \to \infty$. We next consider how to enforce short-term fairness:

Theorem *3:* For N homogeneous nodes with $p_1 = 1$ and $p_2 \geq 1 - \sqrt[N-1]{1-1/M}$, the system is M-short-term fair to all nodes.

Proof: Because D_x is a binomial random variable with parameter $1-p_b$. We have:

$$E[D_x] = \frac{1}{1-(1-p_2)^{N-1}}.$$

Since $p_2 \geq 1 - \sqrt[N-1]{1-1/M}$, $E[D_x] \leq M$. By definition, the system is M-short-term fair to all nodes. ∎

Theorem 3 quantifies how to select p_2 to achieve a certain short-term fairness. In particular, in order to achieve M-short-term fairness, we can choose the following value of p_2.

$$p_2 = 1 - \sqrt[N-1]{1-1/M}. \quad (4)$$

The total throughput becomes a function of M:

$$\rho = \frac{Np_2(1-p_2)^{N-1}}{1+(Np_2-1)(1-p_2)^{N-1}} \quad (5)$$

$$\Longrightarrow \rho = \frac{N(M-1)}{N(M-1) + \frac{1}{1-\sqrt[N-1]{1-\frac{1}{M}}}} \quad (6)$$

Figure 3 plots the total throughput under different short-term fairness constraints (M) as the number of nodes, N is varied along the x-axis. Without sacrificing much throughput, we can achieve very good short-term fairness criteria. For example, if we want the system to be 8-short-term fair, we can achieve a total throughput close to $1/2$ even for large N. Actually, when $N \to \infty$, the total throughput does not collapse to zero. We draw the limits of the throughput in dotted lines under each throughput curve. We will discuss these limits in a later theorem.

Lemma *1:* For any constant $M > 0$, if $p_2 = 1 - \sqrt[N-1]{1-1/M}$, then Np_2 is monotonically decreasing with N.

Proof: Let $\alpha = 1 - 1/M$ and $\beta = \alpha^{\frac{1}{N-1}}$. We have:

$$p_2 = 1 - \sqrt[N-1]{1-1/M} = 1 - \alpha^{\frac{1}{N-1}} = 1 - \beta.$$

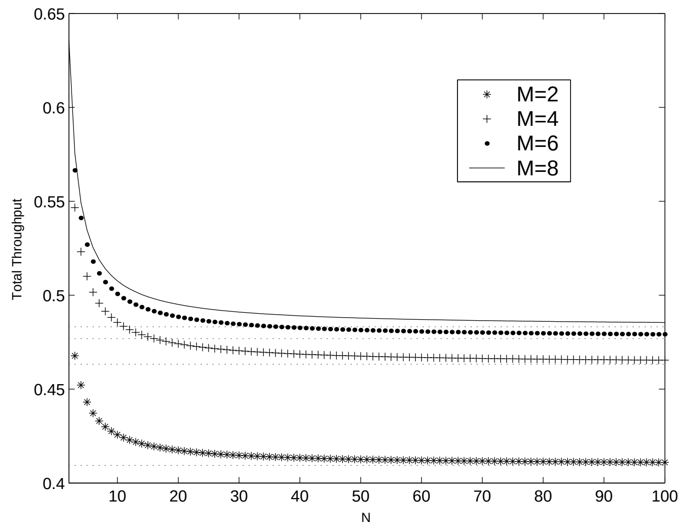

Fig. 3. Throughput under different fairness conditions.

First, p_2 is strictly decreasing in N, because $dp_2/dN = \ln\alpha \frac{\beta}{(N-1)^2} < 0$. Let us define $f(N) = (N-1)p_2$.

$$\begin{aligned} df(N)/dN &= (N-1)[\ln\alpha \tfrac{\beta}{(N-1)^2}] + 1 - \beta \\ &= ((N-1)^{-1}\ln\alpha - 1)\beta + 1 \\ &= (\ln\beta - 1)\beta + 1 \\ &= \beta\ln\beta - \beta + 1 < \ln\beta - \beta + 1 < 0. \end{aligned}$$

The last inequality holds because of the following. Let us define $g(\beta) = \ln\beta - \beta + 1$. $g(\beta)$ is a strictly concave function. $g'(\beta) = 1/\beta - 1$ and $g''(\beta) = -1/\beta^2$. Since $g(1) = 0, g'(1) = 0$ and $g''(1) = -1 < 0$, we see $g(\beta)$ attains its maximum value 0 at $\beta = 1$. Because $\beta < 1$ under our context, the last inequality holds.

Finally, $Np_2 = (N-1)p_2 + p_2$. Therefore, $dNp_2/dN = df(N)/dN + dp_2/dN < 0$. ∎

Theorem *4:* Under any short-term fairness condition $E[D_x] = M$, ρ is lower-bounded by $\frac{-(M-1)\ln(1-\frac{1}{M})}{1-(M-1)\ln(1-\frac{1}{M})}$.

Proof: We choose the value of p_2 in Equation (4) to satisfy the short-term fairness condition. Accordingly, from Equation (6), we have

$$\frac{1}{M-1}\frac{1}{\frac{1}{\rho}-1} = (1 - \sqrt[N-1]{1-\frac{1}{M}})N = \frac{1-\sqrt[N-1]{1-\frac{1}{M}}}{\frac{1}{N}}$$

The right hand side is in the form of $\frac{0}{0}$ as $N \to \infty$. By L'hospital rule,

$$\lim_{N\to\infty} \frac{\ln(1-\frac{1}{M})\sqrt[N-1]{1-\frac{1}{M}}(N-1)^{-2}}{-N^{-2}} = -\ln(1-\frac{1}{M})$$

$$\therefore \lim_{N\to\infty} \frac{1}{M-1}\frac{1}{\frac{1}{\rho}-1} = -\ln(1-\frac{1}{M})$$

$$\Rightarrow \lim_{N\to\infty} \rho = \frac{-(M-1)\ln(1-\frac{1}{M})}{1-(M-1)\ln(1-\frac{1}{M})}$$

By Lemma 1, ρ is monotonically decreasing in N. Therefore, ρ is lower-bounded by $\frac{-(M-1)\ln(1-\frac{1}{M})}{1-(M-1)\ln(1-\frac{1}{M})}$. ∎

Theorem 4 provides the lower bounds for the curves in Figure 3. We draw these limits in dotted lines under each throughput curve. We see, for M to be reasonably large (e.g. $M = 8$), the throughput lower limit is close to $1/2$. Indeed, when M tends to infinity, this limit also tends to $1/2$.

V. Competitive Performance Analysis

In the previous section, we identified the lower bounds of the obtainable throughput among cooperating nodes, even taking into account short-term fairness requirements. In this section, we assume that each node is autonomous and sets its protocol parameters to greedily maximize its own throughput, subject to currently observed conditions. First, let us see how a single node can increase its own throughput by deviating from the cooperative solution. After that, we formulate a constrained optimization problem for each node to maximize its throughput. We construct a *Stackelberg game*[5] which a pair of nodes can play. This game reveals that a *Prisoner's Dilemma*[5] phenomenon can occur.

A. Selfish Behavior in a Cooperative Environment

Suppose N nodes are originally cooperative and use $p_1 = 1$ and $p_2 = 1 - \sqrt[N-1]{1 - 1/M}$ to achieve the maximum M-short-term fair aggregate throughput. In this system, each node x obtains throughput:

$$T_x = \rho/N = (M-1)/[N(M-1) + 1/p_2].$$

If one node deviates from this setting and sets $p_2 = 1$ instead, its throughput increases to

$$T'_x = p_b = (1 - p_2)^{N-1} = 1 - 1/M.$$

Its throughput now equals the probability that no other node is transmitting in each time-slot. Comparing the above two equalities, we have:

$$\frac{T'_x}{T_x} = \frac{N(M-1) + 1/p_2}{M} = N + \frac{1 - Np_2}{Mp_2}$$

Hence, by unilaterally changing p_2 to be 1, a selfish node can usually increase its throughput at least N times (if $Np_2 < 1$). This change sacrifices the throughput of all of the other nodes, which no longer obtain any throughput.

B. Stackelberg Game

We have shown that a single selfish node can increase its own throughput in a setting where all other nodes are cooperative. We now explore what happens when multiple nodes set their parameters in a greedy fashion. Here, let us consider a network that consists only of two selfish nodes x and y, each of which wants to maximize its own throughput. In addition, we assume that each have budget constraints $C_x \le B_x$ and $C_y \le B_y$ respectively. C_x and C_y are the costs of both nodes as defined in Equation (3). $B_x, B_y \in (0, 1]$ are two budget constants which physically restrict the average number of packets the node can transmit in each time-slot. We impose these budget constraints in order to model the nodes in a wireless ad-hoc network or a sensor network. Because nodes in these networks are very sensitive to the power consumption, and transmitting packets consumes a lot of battery power. We model the competition between these two nodes using a *Stackelberg game*[5], in which a "leader" chooses a strategy (i.e. the transmitting probabilities in both the Free State and the Backlogged State) and then a "follower", informed of the leader's choice, chooses a strategy. We formulate a non-cooperative Stackelberg game as follows:

Players:	The leader node x and the follower node y.
Strategy:	$S^x = \{p_1^x, p_2^x\}$ for x; $S^y = \{p_1^y, p_2^y\}$ for y.
Payoff:	T_x and T_y for x and y respectively.
Game rule:	x decides $\{p_1^x, p_2^x\}$ first.
	y decides $\{p_1^y, p_2^y\}$ after knowing $\{p_1^x, p_2^x\}$.

Follower's Problem:

The follower y is given the leader's chosen parameters. It then simply sets its own parameters to maximize its own throughput. More formally, for any given $\widetilde{S^x}$, the follower node y solves:

$$\hat{S^y}(\widetilde{S^x}) = arg\ max\ \ T_y(\widetilde{S^x}, \hat{S^y})$$
$$\texttt{Subject to}: C_y(\widetilde{S^x}, \hat{S^y}) \le B_y.$$

Leader's Problem:

The leader knows that the follower will choose its parameters to greedily maximize its own throughput. Therefore, the leader must choose its protocol parameters that will maximize its throughput, given the follower will subsequently choose its own parameters to maximize its throughput. More formally, the leader node x solves:

$$\hat{S^x} = arg\ max\ \ T_x(\hat{S^x}, \hat{S^y}(\hat{S^x}))$$
$$\texttt{Subject to}: C_x(\hat{S^x}, \hat{S^y}(\hat{S^x})) \le B_x.$$

In order to solve this Stackelberg game, we first solve the follower's problem for every possible strategy taken by node x. Thus, we obtain the best response strategy of y as a function of node x's strategy. After that, the leader decides its optimal strategy according to node y's best response strategy. This procedure is often referred to as *backwards induction*[4][17]. The according game solution is often referred to as a Stackelberg equilibrium.

C. Three Stackelberg Equilibrium Regions

We solve the above Stackelberg game for nodes who have the same budget constraints, which means $B_x = B_y$.

Figure 4 shows the throughput and costs of both players in Stackelberg equilibrium. X-axis represents the budget constraint for both players. The change in the throughputs as a function of the budget behaves differently in three different regions:

1) When the budget is less than $1/3$, both players achieve the same throughput. They use up their budgets. The

[4]Backward induction is actually a more general procedure to identify the Subgame Perfect Nash Equilibra in any finite dynamic game with perfect information.

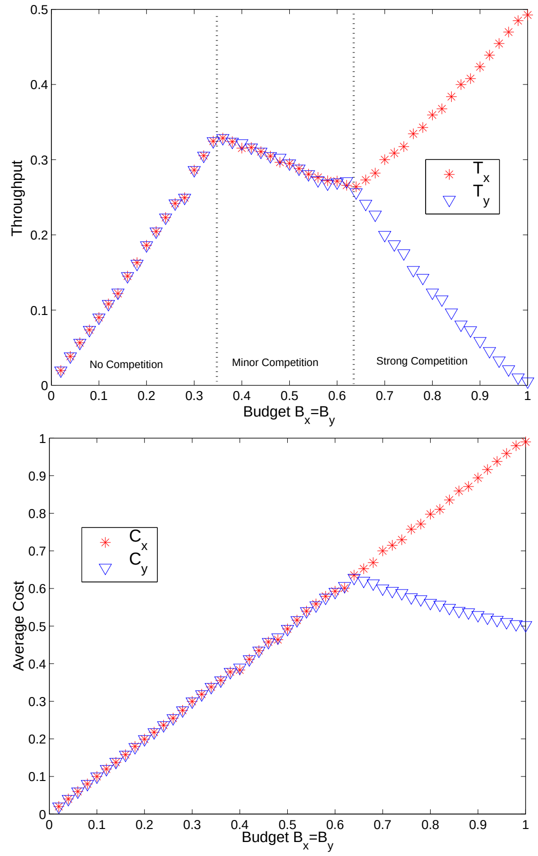

Fig. 4. Throughput and cost in Stackelberg equilibrium.

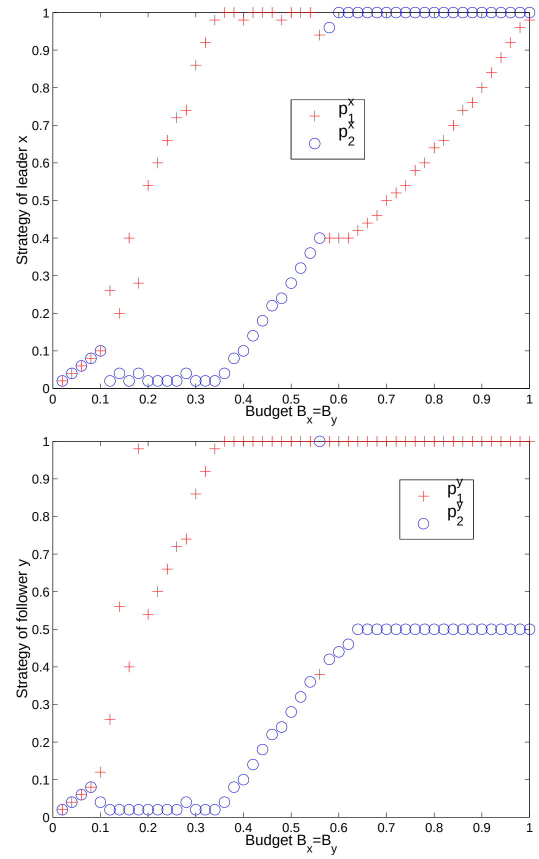

Fig. 5. Strategies in Stackelberg equilibrium.

throughput is mainly limited by the budget constraints, but not the competition between these two players. Increasing the budget increases their achieved throughputs.

2) When the budget is between $1/3$ and $2/3$, both players again choose similar strategies and achieve similar throughputs. However, increasing both players' budgets decreases each player's throughput. In this region, the throughput is limited by both the budget constraints and the competition between these two players. Note that because similar strategies are chosen, it does not matter (to a node) whether it is chosen to be the leader or the follower.
3) When the budget is more than $2/3$, the leader can select parameters that give it a larger fraction of the throughput. The follower, still wishing to maximize its own throughput, actually becomes less aggressive. In this region, it is clearly preferable to be the leader.

Figure 5 shows the strategies of both players in Stackelberg equilibrium. In the first two solution regions, both players use similar strategies. When the budgets are close to $1/3$, the greedy strategies selected by the players are similar to what would be selected by cooperative players, and the aggregate throughput approaches $2/3$. As the budgets are further increased, the nodes' additional greed increases the contention on the line and the rate of interference becomes significant. When the budgets exceed $2/3$, the leader's strategy quickly changes. It sets $p_2 = 1$, which means if a transmission fails in a slot, it attempts retransmission during the next slot. This makes sense intuitively because the follower, attempting to maximize its own throughput with its confined budget, must back off with high probability after a collision, and the "safest" time for the follower to transmit will be following a previous successful transmission. Hence, the follower sets $p_1 = 1$, since it can only be in the Free State when the leader is also in the Free State, and it can only successfully transmit when the leader is in the Free State.

D. *Prisoner's Dilemma*

From the above Stackelberg game, the leader node x can achieve a throughput of at most $T'_x = 1/2$. Note that this throughput is better than it can gain in a cooperative environment ($T_x < 1/3$). We continue to assume that both nodes will decide their strategies simultaneously.

Let us consider three budget scenarios and the corresponding strategies that would be played in the Stackelberg game:

- Low budget region: $B_x = B_y = 0.34$.
  Strategy $S_C = S^x = S^y = \{p_1 = 0.98, p_2 = 0.02\}$.
- Medium budget region: $B_x = B_y = 0.5$.
  Strategy $S_M = S^x = S^y = \{p_1 = 1, p_2 = 0.28\}$.
- High budget region: $B_x = B_y = 0.8$.
  Strategy $S_L = S^x = \{p_1 = 0.64, p_2 = 1\}$,
  and $S_F = S^y = \{p_1 = 1, p_2 = 0.5\}$.

Strategy S_C in the lower budget region is similar to the strategy played by the nodes in a cooperative environment.

Strategy S_M, which is more aggressive than S_C, is played by both the leader and the follower in the middle budget region. Finally, S_F and S_L are the respective strategies of the leader and the follower in the high budget region.

Now, let us consider two situations where two nodes enter into a common medium and must choose their parameters to maximize their individual throughputs without knowing what their opponent chooses to do:

To Cooperate or to Compete?

Consider a game in which two nodes, each with a budget of 0.5, share a common medium. These nodes must decide whether they will cooperate or behave in a greedy manner (i.e., should the node set its parameters according to S_C or S_M?)

The throughput of both players can be depicted by the following table.

	S_C	S_M
S_C	(0.3246,0.3246)	(0.0034,0.9288)
S_M	(0.9288,0.0034)	**(0.2951,0.2951)**

The most efficient solution is at (S_C, S_C). However, a selfish node will note that whichever strategy its opponent chooses, its throughput will be increased by choosing S_M. Here, we see a typical Prisoner's Dilemma[5]. Although from a global perspective, both players know the best solution is (S_C, S_C), from any hypothetical local point, strategy S_M should always be played. This is because, for any fixed strategy by the opponent, choosing S_M is always better than choosing S_C. Strategy S_M is called the *dominating strategy*[5] for both players and the solution (S_M, S_M) is the unique *Nash equilibrium*[5] solution of this game.

To Lead or to Follow?

In the second game, we assume that nodes have budgets in the third region. As before, the nodes are better off playing a greedy strategy. However, now the nodes must also decide whether to choose the leader's strategy or the follower's strategy.

	S_F	S_L
S_F	(0.25,0.25)	**(0.1233,0.3595)**
S_L	**(0.3595,0.1233)**	(0,0)

Here, a node's strategy is not clear. A node is always better off choosing the opposing strategy of its competitor. Choosing the follower strategy is more conservative. A throughput of at least 0.1233 is ensured, but the throughput can be at most 0.25. If the leader strategy is chosen, a throughput of 0.3595 is possible, but a throughput of 0 is also a possible outcome. Interestingly, this game has two symmetric Nash equilibrium solutions, which are (S_F, S_L) and (S_L, S_F).

VI. Selfish Behavior Detection and Prevention

In the previous section, we used non-cooperative games of two nodes to show that selfish behavior of nodes deteriorate the overall throughput obtained across the transmission medium, as well as that of the individual nodes. In this section, we discuss how cooperating nodes can identify and prevent selfish behavior in a general N-node system.

A. Transmitting is a Dominating Strategy

Consider any node i at any time-slot t. If it attempts to transmit, the probability of success is

$$\Pi_{j \neq i}(1 - p_j(t))$$

where $p_j(t)$ is the transmitting probability of node j at that time-slot. Without any budget constraint, if node i were greedy, it would always be better for node i to transmit a packet during every time-slot in order to maximize its throughput.

But if node i transmits packet in every time-slot, other nodes transmission attempts will always fail. Over time, this phenomenon is easily observed. Here, we consider how cooperative nodes can alter their parameters if their perceived throughputs are too small in such a way that selfish nodes become "encouraged" to set their parameters in a cooperative manner.

B. Selfish Behavior Detection

Theorem *5: For a M-short-term fair cooperative environment, where each node uses $p_1 = 1$ and $p_2 = 1 - \sqrt[N-1]{1 - 1/M}$, the successful rate defined by T_x/C_x for any node x is lower-bounded by $\frac{(M-1)(M-2)}{(M-1)(M-2)+1}$.*

Proof:

$$\frac{T_x}{C_x} = \frac{NT_x}{NC_x} = \frac{\rho}{NC_x}$$

NC_x is equals to the total average cost for all nodes. Suppose when all N nodes are in backlogged state. Let Q to be the number of nodes which decide to transmit in a time-slot. Therefore, $Pr\{Q = i\} = q_i = \binom{N}{i} p_2^i (1 - p_2)^{N-i}$ and Q is a binomial random variable with parameter p_2 and N.

$$\begin{aligned} NC_x &= \rho + (1-\rho)\textstyle\sum_{j=2}^{N} j q_j/(1-q_1) \\ &= \rho + (1-\rho)(E[Q] - q_1)/(1-q_1) \\ &= \rho + (1-\rho)(Np_2 - q_1)/(1-q_1) \end{aligned}$$

Since $q_1 = Np_2(1-p_2)^{N-1} = Np_2(1 - \frac{1}{M})$, and $\frac{1}{\rho} - 1 = \frac{1}{M-1}\frac{1}{Np_2}$.

$$\begin{aligned} & T_x/C_x = \rho/[\rho + (1-\rho)(Np_2 - q_1)/(1-q_1)] \\ \iff & \tfrac{1}{T_x/C_x} - 1 = (\tfrac{1}{\rho} - 1)(Np_2 - q_1)/(1-q_1) \\ \iff & \tfrac{1}{T_x/C_x} - 1 = \tfrac{1}{M-1}\tfrac{1}{Np_2}(Np_2 - Np_2(1 - \tfrac{1}{M}))/(1-q_1) \\ \iff & \tfrac{1}{T_x/C_x} - 1 = \tfrac{1}{M-1}(\tfrac{1}{M})/(1-q_1) \\ \iff & \tfrac{1}{T_x/C_x} - 1 = \tfrac{1}{M-1}(\tfrac{1}{M})/(1 - Np_2(1 - \tfrac{1}{M})) \end{aligned}$$

By Lemma 1, Np_2 is monotonically decreasing in N. When $N = 2$, $Np_2 = 2/M$ is the maximum for $N > 1$. Substitute Np_2 with $2/M$, we have:

$$\begin{aligned} & \tfrac{1}{T_x/C_x} - 1 = \tfrac{1}{M-1}(\tfrac{1}{M})/(1 - Np_2(1 - \tfrac{1}{M})) \\ \Longrightarrow & \tfrac{1}{T_x/C_x} - 1 \leq \tfrac{1}{M-1}(\tfrac{1}{M})/(1 - \tfrac{2}{M}(1 - \tfrac{1}{M})) \\ \Longrightarrow & \tfrac{1}{T_x/C_x} - 1 \leq \tfrac{1}{M-1}/(M - 2(1 - \tfrac{1}{M})) < \tfrac{1}{M-1}\tfrac{1}{M-2} \\ \Longrightarrow & T_x/C_x > [(M-1)(M-2)]/[(M-1)(M-2)+1] \end{aligned}$$

■

Theorem 5 gives the guideline for cooperative nodes to detect the existence of any selfish node. A fraction of at least $\frac{(M-1)(M-2)}{(M-1)(M-2)+1}$ of a cooperative node's transmissions should be successful. For instance, when M equals 8, this average success rate lower-bound is $\frac{42}{43}$. When M is larger, the success rate is even higher. In practice, each node can measure this quantity to infer if there is any selfish node in the system.

C. Selfish Behavior Prevention

In order to prevent selfish behaviors in the system, all cooperative nodes should activate a new strategy using a $p'_2 > p_2$ that reduces the throughput of the selfish node beneath what it would have been had when it used the cooperative parameters. Knowing that such a reduction will occur gives the selfish nodes the necessary incentive to remain cooperative.

Suppose after all cooperative nodes activate the new strategy p'_2, the selfish node obtains throughput T'_x. T'_x has to be less than ρ/N, the fair share throughput gained in a cooperative environment.

$$\begin{aligned} & T'_x = (1-p'_2)^{N-1} < \rho/N \\ \Longleftrightarrow \quad & 1 - p'_2 < \sqrt[N-1]{\rho/N} \\ \Longleftrightarrow \quad & p'_2 > 1 - \sqrt[N-1]{\rho/N} \end{aligned}$$

From Theorem 4, we know that ρ is lower-bounded by $1/2$. Hence, we can substitute in $1/2$ for ρ when calculating p'_2 as an approximation.

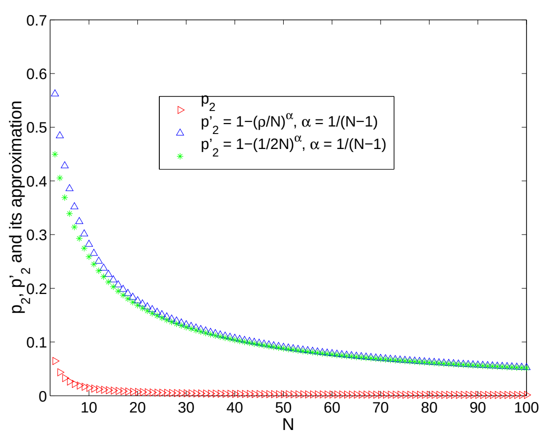

Fig. 6. Cooperative p_2, selfish preventive p'_2 and its approximation.

Figure 6 shows the cooperative strategy p_2 and the selfish prevention strategy p'_2. We see $1 - \sqrt[N-1]{1/2N}$ is a good approximation for the lower-bound for p'_2.

VII. Adversary Model Analysis

All previous scenarios assume that each node, whether cooperative or selfish, is interested in maximizing its own throughput. In this section, we consider an attacking node whose goal is to use its restricted budget to minimize the throughput of the other nodes in the system, i.e., to cause as many of its packets to collide with what would otherwise be a successful transmissions. We first discuss how much damage it will cost if an attack uses a random (stateless) attack. Next, we formulate this attack model as another Stackelberg game.

A. Pure Random (Stateless) Attack

If an attacking node is able to transmit a packet in every slot, it can clearly prevent any transmission from being successful. We assume that the adversary node has a budget $B \in (0, 1]$, allowing it to transmit in at most a fraction B of the slots.

Definition *2:* An adversary node uses p-pure random attack if it transmits a packet in each time-slot independently with probability p.

By Definition 2, an adversary node with a budget B can use a p-pure random attack for any $p \le B$. We can imagine that p-pure random attack for a communication channel is identical to a lossy channel where a packet is lost with probability p.

Theorem *6:* Suppose there are two nodes x and y in the system. If node x is an adversary node which uses p-pure random attack, then regardless of the strategy of player y, its throughput T_y is equal to $(1-p)C_y$.

Proof: Substitute p_1^x and p_2^x with p in the corresponding throughput for y as in Equation (2). We have:

$$\begin{aligned} T_y &= (\pi_1, \pi_3, \pi_2, \pi_4) \begin{pmatrix} p_1^y(1-p_1^x) \\ p_1^y(1-p_2^x) \\ p_2^y(1-p_1^x) \\ p_2^y(1-p_2^x) \end{pmatrix} \\ &= (1-p)(\pi_1, \pi_3, \pi_2, \pi_4) \begin{pmatrix} p_1^y \\ p_1^y \\ p_2^y \\ p_2^y \end{pmatrix} \end{aligned}$$

Since the corresponding cost function for y as in Equation (3), we have:

$$C_y = (\pi_1, \pi_3, \pi_2, \pi_4) \begin{pmatrix} p_1^y \\ p_1^y \\ p_2^y \\ p_2^y \end{pmatrix}$$

Therefore, $T_y = (1-p)C_y$. ∎

Theorem 6 formalizes the intuitive result that a p-pure random attack reduces the capacity to be $1-p$ of the original capacity. Interestingly and initially countering preliminary intuition, if we have more than one cooperative node, the damage caused by a p-pure random attack is often larger than a factor of $1-p$:

Theorem *7:* Suppose originally there are N homogeneous nodes which use $p_1 = 1$ and $p_2 < 1/N$ in the system. They achieve an aggregate throughput ρ. If an adversary node joins the system and uses p-pure random attack, then the aggregate throughput of the N cooperative node is less than $(1-p)\rho$.

Proof: Before the adversary node comes into the system, we can model the system as in Figure 2. The transition probabilities are $p_b = (1-p_2)^{N-1}$ and $p_a = Np_2(1-p_2)^{N-1}$. After the adversary node comes, we define the corresponding transition probabilities to be p'_b and p'_a. Because a successful packet from a normal node happens only if the adversary node does not transmit, we have $p'_b = (1-p)p_b$ and $p'_a = (1-p)p_a$.

From Equation (5),

$$\rho = \frac{Np_2(1-p_2)^{N-1}}{1 + (Np_2 - 1)(1-p_2)^{N-1}}$$

The new throughput ρ' is

$$\rho' = \frac{p'_a}{1 - p'_b + p'_a} = \frac{Np_2(1-p_2)^{N-1}}{\frac{1}{1-p} + (Np_2 - 1)(1-p_2)^{N-1}}$$

Therefore,

$$\frac{\rho'}{\rho} = \frac{1 + (Np_2 - 1)(1-p_2)^{N-1}}{\frac{1}{1-p} + (Np_2 - 1)(1-p_2)^{N-1}} < 1 - p$$

The last inequality holds if $p_2 < 1/N$. ∎

An explanation of this result is as follows: as more nodes participate in the cooperative process, the expected number of slots between transmissions in the Backlogged State grows at a faster rate than the expected number of slots between transmissions in the Free State. A random seeding of losses forces more nodes to spend more time in the Backlogged State, and as a result, each node attempts fewer transmissions over time, yet still loses a fraction p of the attempts to the random loss process.

B. Adversary Stackelberg Game

Now, let us compute the reduction in throughput that an attacking node can cause if it maximizes its attack power under a 2-state system. As in section V-B, we introduce a Stackelberg game in this section. The difference between the previous model and this model is that we assume the leader node x is the attacker and its sole objective is to minimize the throughput of node y. Because the leader always has the advantage over the follower, making the attacking node the leader maximizes its potential for damage. We still assume that node x and y have budget constraints: $C_x \leq B_x$ and $C_y \leq B_y$ respectively. The adversary Stackelberg game can be formally described as follows:

Player:	The leader node x and the follower node y.
Strategy:	$S^x = \{p_1^x, p_2^x\}$ for x; $S^y = \{p_1^y, p_2^y\}$ for y.
Payoff:	$-T_y$ and T_y for x and y respectively.
Game rule:	x decides $\{p_1^x, p_2^x\}$ first.
	y decides $\{p_1^y, p_2^y\}$ after knowing $\{p_1^x, p_2^x\}$.

Follower's Problem:

For any given $\widetilde{S^x}$, the follower node y solves:

$$\hat{S^y}(\widetilde{S^x}) = arg\ max\ \ T_y(\widetilde{S^x}, \hat{S^y})$$
$$\texttt{Subject to}: C_y(\widetilde{S^x}, \hat{S^y}) \leq B_y.$$

Leader's Problem:

The leader node x solves:

$$\hat{S^x} = arg\ min\ \ T_y(\hat{S^x}, \hat{S^y}(\hat{S^x}))$$
$$\texttt{Subject to}: C_x(\hat{S^x}, \hat{S^y}(\hat{S^x})) \leq B_x.$$

C. Two Stackelberg Equilibrium Regions

By backward induction, we solve the above adversary Stackelberg game for nodes who have the same budget constraints, i.e., $B_x = B_y$. In the upper part of Figure 7, we plot the throughput of the follower (non-attacking) node y when x chooses the optimal 2-state attacking strategy. We also plot

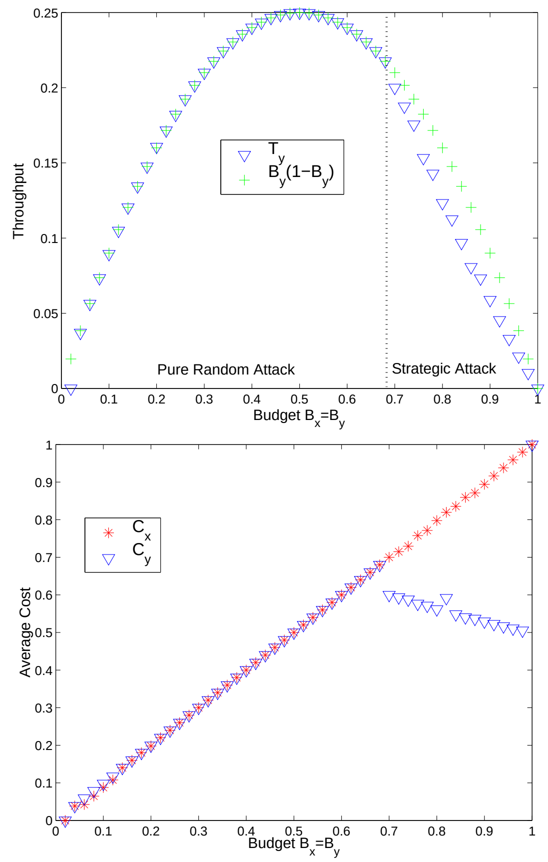

Fig. 7. Throughput and cost in (adversary) Stackelberg equilibrium.

the curve $B_y(1 - B_y)$, which gives the throughput of node y when the attacker uses a p-pure random attack with $p = B_y$. In the lower part of Figure 7, we show the costs incurred by both players. We identify two regions in the Stackelberg equilibrium solutions:

1) When the budget is less than $2/3$, both players use up their budgets. The throughput of player y when attacked by the optimal 2-state attacker is identical to its throughput when attacked by a p-pure random attacker.
2) When the budget is larger than $2/3$, player y's throughput is slightly but observably lower when attacked by the optimal 2-state attacker than when attacked by the p-pure attacker.

Intuitively, the attacking node will always use up its budget to attack. But surprisingly, a strategic, 2-state attack cannot do better than pure random attack if the adversary node does not have a budget larger than $2/3$. When the budget is larger than $2/3$, the 2-state attack is only slightly more effective.

D. Random Attack Vs. Strategic Attack

We show the strategy solutions of both players in Figure 8. We find that the strategies played in the two budget regions are quite different.

Not surprisingly, when the budget is less than $2/3$, the attacking node uses the pure random strategy $p_1^x = p_2^x = B_x$. Theorem 6 explains why the throughput T_y is so close to curve $B_y(1 - B_y)$ in the lower budget region. Actually, player

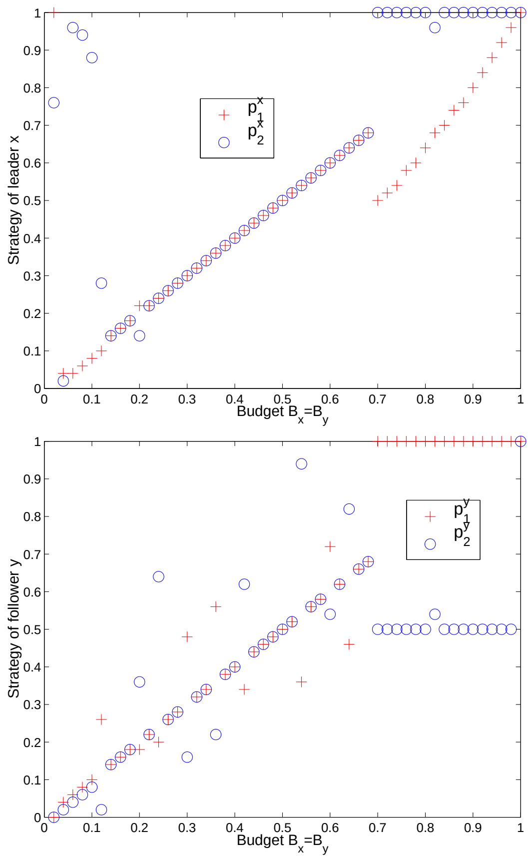

Fig. 8. Strategies in (adversary) Stackelberg equilibrium.

y has multiple strategies to maximize its throughput. But all these strategies use up the budget B_y. Therefore, although the strategies played by node y seem to be irregular, node y always gains a throughput which is close to $B_y(1 - B_y)$.

After comparing the strategies played by both nodes in the larger budget region with those used by two non-cooperative, non-attacking nodes in Figure 5, we notice that they are strikingly similar. This means that an attacking node x chooses a strategy very similar to what is chosen by a node who wishes to greedily maximize its own throughput. Of course, node y would therefore use the same response strategy.

In conclusion, if bandwidth requirements/capabilities are low, an attacker cannot do much better than attacking at random points in time. If the bandwidth requirements and capabilities are high, then an attacker behaves similarly to a node seeking to greedily maximize its own throughput.

VIII. Conclusion

In this paper, we generalize the slotted-Aloha protocol to a general two-state process. We construct a Markov model for this generalized two-state protocol. We find that if all nodes cooperate in an effort to maximize the aggregate throughput, an aggregate throughput of at least $1/2$ can be achieved regardless of the number of nodes competing for bandwidth. If all nodes are selfish and greedily attempt to maximize their own individual throughputs, a situation similar to the traditional Prisoner's Dilemma arises. Specifically, for a two-node system with budget constraints, the solution has the following features: (1) When each node's transmitting budget is extremely limited, a greedy strategy maximizes each individual node's throughput as well as the aggregate system throughput. (2) When each node's transmitting budget is in a middle range, a greedy strategy produces a local maximum throughput, but a cooperative strategy would have produced a higher throughput. (3) When each node's budget is in the upper range, a node's greedy strategy depends heavily on what its competitors choose to do. Finally, we showed that attacking nodes with limited budgets can do little better than a random attack, and nodes with large budgets should behave like their greedy counterparts.

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