# An Analysis of Generalized Slotted-Aloha Protocols

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**Abstract**

Aloha and its slotted variation are commonly deployed Medium Access Control (MAC) protocols in environments where multiple transmitting devices compete for a medium, yet may have difficulty sensing each other's presence (the "hidden terminal problem"). Competing 802.11 gateways, as well as most modern digital cellular systems, like GSM, are examples. This paper models and evaluates the throughput that can be achieved in a system where nodes compete for bandwidth using a generalized version of slotted-Aloha protocols. The protocol is implemented as a two-state system, where the probability that a node transmits in a given slot depends on whether the node's prior transmission attempt was successful. Using Markov Models, we evaluate the channel utilization and fairness of these types of protocols for a variety of node objectives, including maximizing aggregate throughput of the channel, each node greedily maximizing its own throughput, and attacker nodes attempting to jam the channel. If all nodes are selfish and greedily attempt to maximize their own throughput, a situation similar to the traditional Prisoner's Dilemma arises. Our results reveal that under heavy loads, a greedy strategy reduces the utilization, and that attackers cannot do much better than attacking during randomly selected slots.

## I. Introduction

In many communication networks, the communication medium is often shared by multiple users who must compete for access. In Ethernet [1], nodes use CSMA/CD [2], [3] as a MAC protocol. In order to reduce the probability of collisions, each node implements CSMA/CD, sensing the medium to ensure its availability prior to transmitting. However, for wireless ad-hoc networks or sensor networks, carrier sensing may not be effective. This is because nodes may not be able to sense one anothers' presence, yet their transmissions may still interfere. Ad hoc networks, sensor networks, and competing "hotspot" 802.11 gateways are examples where this so-called "hidden terminal problem" occurs.

The Aloha protocol [4] is a fully decentralized medium access control protocol that does not perform carrier sensing. The subsequent slotted-Aloha [5] protocol was introduced to improve the utilization of the shared medium by synchronizing the transmission of devices within time-slots. Today, various forms of slotted-Aloha protocols are widely used in most of the current digital cellular networks, such as the Global System for Mobile communications (GSM)[1].

In this work, we consider a generalization of the slotted-Aloha protocol. Like slotted-Aloha, the decision to transmit within a slot has a random component. However, in traditional slotted-Aloha, the user continues transmission in subsequent slots until a subsequent collision. In our generalized version, the user may cease transmitting with some fixed (non-zero) probability. We model a system of $N$ users implementing this generalized protocol with tunable parameters via Markov Models that allow us to measure the rate at which nodes attempt to transmit packets (cost), and their rates of success (throughput). In parts, we impose budget constraints that restrict the nodes' costs, such that the fraction of slots within which a node attempts transmissions is bounded. In practice, these additional constraints may be due to energy constraints, or a bandwidth constraint placed on the network application.

This generalized version of slotted-Aloha is worth studying for several reasons. First, it is derived from a protocol that is commonly used today. In slotted-Aloha, a node is in one of two states – backlogged or successful. A natural generalization is to assign (re)transmission probabilities for each of the states. Second, introducing additional states to describe the level of the backlog or number of successive collisions would move the protocol towards binary exponential backoff protocols, e.g., 802.11 family of MAC protocols; however, this generalized version of slotted-Aloha retains the simplicity and elegance of the original Aloha approach. Third, we will show that the generalized versions can outperform the original version, both in terms of aggregate throughput, as well as the ability to cope with malicious users. Fourth, by using this generalized slotted-Aloha protocol, we provide a framework to study the user behaviors in cooperative, competitive and adversarial environments.

We begin by exploring an environment where $N$ users cooperate and set the protocol parameters to maximize the total system throughput while sharing the bandwidth evenly. We find that the throughput is bounded by $N/(2N-1)$ and that to achieve this utilization, users who gain access to the channel must transmit over a large number of consecutive slots. We then explore how throughput decreases as "short-term fairness" is more strictly enforced, reducing the expected number of consecutive slots.

Next, we consider selfish users who wish to maximize their own throughput, perhaps at the expense of the nodes against whom they compete. We evaluate this setting as a game, where some node strategically chooses its parameters and another node subsequently modifies its parameters in response to maximize its own throughput. We find that performance of the protocol depends on nodes' budgets, and takes on three distinct types of behavior. When nodes' budgets are low, the aggressive and

[1] In the GSM network, the control channels of the TDM channels use slotted-Aloha.

greedy strategy is optimal. When nodes' budgets fall within a medium range, all nodes achieve the same throughput in a unique equilibrium, but the throughput is less than what would be obtained in a cooperative setting. When nodes' budgets fall within the highest range, multiple equilibria exhibit that the throughput achieved by a node depends on when it took its turn in the game and that the Prisoner's Dilemma can occur. We develop an additional enhancement to the protocol that can be implemented by cooperative nodes which will encourage greedy users to tune their protocol parameters to match those of a cooperative node, maximizing the aggregate throughput.

Last, we consider an attacking node that, with a limited budget, seeks to minimize the throughput of the other nodes in the system. We show that when the attacker's budget is small, selecting random slots (i.e., via a Bernoulli process) is optimal. When the budget is large, the optimal strategy is to mimic a greedy user. Our analysis provides insights on the limits of success a jammer can have in disrupting a slotted-Aloha like network.

We summarize the main contributions of this paper as follows:

1) We formulate different user behaviors under a generalized slotted-Aloha protocol where users make transmission decisions using a two-state system.
2) We identify throughput bounds for a system of cooperative users and explore the trade-off between user throughput and short-term fairness.
3) Under non-cooperative/selfish behavior of the users, we identify a Prisoner's Dilemma phenomenon and propose methods to detect and prevent nodes from acting selfishly without regard for other nodes' throughput.
4) Under adversarial behavior of one user, we measure the maximum possible deterioration of the system and try to understand the behavior of an attacker.

We organize our paper as follows. In Section II, we review related work. In Section III, we motivate the protocol and construct a Markov Model for the generalized slotted-Aloha protocol. In Section IV, we measure the system throughput in a cooperative environment where users want to maximize the total throughput of the system. In Section V and VI, we evaluate both the aggregate and individual user throughput where selfish users exist in the system. We formulate a Stackelberg game [6] and identify a Prisoner's Dilemma situation in Section V, and in Section VI present strategies that cooperative nodes can implement to detect and prevent selfish user behaviors. In section VII, we explore a system in which an attacker tries to minimize the throughput of the remaining nodes. Section VIII concludes.

## II. **Related Work**

The Aloha protocol and its slotted version have been studied for decades from the early seventies. Because slotted-Aloha exhibits an instability as the number of transmitting nodes increases [7], [8], [9], [10], [11], [12], early research focused on stabilizing the Aloha protocols [12], [13]. Rivest [13] proposed a pseudo-Bayesian algorithm, which utilizes feedback to estimate the number of current backlogged nodes in the system, to stabilize Aloha. Later, even many performance evaluations of the Aloha protocols were accompanied with dynamic controls [14], [9] to stabilize the systems. However, today's networks often implement an admission control procedure to limit the number of simultaneous users in the system at any time. In this sense, the system itself is stable in terms of users.

In this work, we focus on the performance of stable slotted-Aloha type systems, where only a finite number of users will access the shared medium simultaneously. Early work on slotted Aloha with finite number of users can be found in [7]. Not only does our work evaluate the throughput bounds for a finite slotted-Aloha type system, but it also considers the performance of individual users under different types of user behavior. Many prior work [15], [16] have shown that users always have incentive not to follow the designed protocol (not backoff) in order to achieve higher throughput. Consequently, Game Theory [6], [17] applies here.

Recent work using Game Theory to analyze user behavior in MAC protocols and wireless ad-hoc networks can be found in [18], [19], [15] and [20], [21] respectively. More specifically, game-theoretical analysis of the Aloha protocols can be found in [22], [23][2], [24], [25], [26], [27].

MakKenzie and Wicker's work [24], [25] discussed the stability of slotted-Aloha with selfish user behavior and perfect information. Our work is different in three ways. First, we focus on performance (attainable throughput) instead of stability. In terms of data backlog at the users, we consider scenarios of *elastic transfers*, where users always have data to send and utilize whatever bandwidth is available, and hence classical stability results do not apply to our analysis. Second, we assume that nodes do not know the number of transmitting nodes a particular slot a priori, and know only whether or not their transmission succeeded after the fact. Third, we consider cooperative and attacking strategies in addition to selfish strategies.

Jin and Kesidis's work [26] discusses the equilibra of a non-cooperative game for Aloha protocols. In their non-cooperative game formulation, each user only uses one transmitting probability (i.e., always in a backlogged state). Moreover, utility functions and payments are specified for each user. Our work, on the other hand, formulates generalized slotted-Aloha protocol that considers the Markovian decisions depending on whether the most recent transmission is a success (in a Free State) or a failure (in a Backlogged State). Also, we do not impose any payment on the users.

[2]The citation is our prior work in ICDCS. This paper extends the ICDCS paper and provides complete proofs for all theorems. It also includes new contents such as optimality of performance solution (pp.5–7) and selfish behavior detection and prevention (pp.10–11).

Altman et al. [27] consider slotted-Aloha systems as both cooperative and non-cooperative games with partial information. Their work assumes that there are a finite number of bufferless sources. The arrival of packets to each source follows a Bernoulli process. As in typical slotted-Aloha, users only control the backlog probability in both systems. In our work, we assume saturated arrivals (*elastic transfer*) where each user always has packets to transmit. Our users' strategies are more broad, because users are permitted to choose a non-zero probability to back off even their previous transmissions are successful. In addition, we analyze an adversarial game where an attacker who wants to minimize other users' throughput appears in the game.

## III. **Protocol Description and Model**

In this section, we describe a generalized slotted-Aloha MAC protocol and construct a Markov Model from which its throughput can be measured. We overview the original slotted-Aloha protocol as follows:

1) Time is divided into slots, and each node can attempt to send one packet in each time-slot.
2) If a node has a new packet to send, it attempts transmission during the next time-slot.
3) If a node successfully transmits one packet, it can transmit a new packet in the next time-slot.
4) If a node detects a collision, it retransmits the old packet in each subsequent time-slot with probability $p$ until the packet is successfully transmitted. Returning to step 3 after a successful transmission.

The slotted-Aloha protocol described above can be implemented as a two-state system, where the state maintains the outcome of the previously attempted transmission. A node is in its *Free State* if the most recent transmission from that node is a success; otherwise, the node is in its *Backlogged State*. In the Free State, a node transmits during the next slot with probability $1$, and in the Backlogged State, it transmits during the next slot with probability $p$. Our generalization of the above protocol is to allow a node to vary the probability with which a node transmits a packet when it resides within the Free State. Later, we will see that this generalization enables us to model selfish as well as malicious behavior of users.

Our evaluation will consider a network of $N$ contentious nodes, which always have backlog data packets to transfer and choose whether or not to transmit in each time-slot. Since we focus on nodes that always have backlogs, we analyze the performance of a congested system in the worst case. We assume that data packets are fragmented into lengths that can be transmitted within a time-slot. We assume that nodes are able to coordinate slot transmission times and can estimate the number of nodes $N$ with which they compete for bandwidth. However, because nodes' transmissions may interfere but cannot be recovered, methods to prevent slot contention that require explicit communication and coordination among the competing members (e.g., TDMA, RTS-CTS) cannot be used. In practice, many real systems implement admission control mechanisms which constraint the number of users in the system. We might be able to rely on a centralized authority to broadcast the number of competing users (this information can be embedded in beacon messages).

Each node $x$ can tune its protocol using two parameters: $p_1^x$, the transmitting probability in the Free State, and $p_2^x$, the transmitting probability in the Backlogged State. Given $N$ and the transmitting probabilities for each of the nodes in each of the states, we can compute the following performance metrics: $T_x$, the *throughput* of node $x$, which is the fraction of slots within which $x$ successfully completes a transmission and is the only device to attempt transmission; and $C_x$, the *cost* for node $x$, which is the fraction of slots within which $x$ attempts transmission (regardless of whether the transmission fails or succeeds).

Each node's decision to transmit within a particular slot depends only on the outcome of its previous attempt (success or failure), and does not depend on the state of other nodes. Hence, this protocol can be easily implemented in a distributed manner. Moreover, each node's decision is in fact Markovian, as it depends only on the previous attempt's outcome.

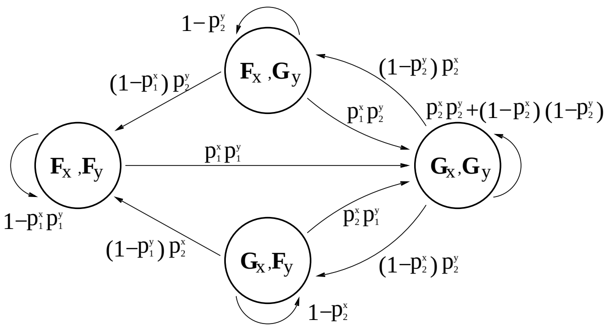



Fig. 1. Two-node Markov Chain.

Figure 1 shows the state transition diagram for a two-node system with node $x$ and $y$. $F_x$ and $G_x$ represent that node $x$ is in a Free State and a Backlogged State respectively. A system for $N$ nodes is easily modeled by as a Markov Model where the chain would consist of $2^N$ states. By numbering the states $(F_x, F_y), (F_x, G_y), (F_y, G_x), (G_x, G_y)$ to be $1, 2, 3, 4$, the transition matrix for a two-node Markov Model is:

$$P = \left( \begin{array}{cccc} 1-p_1^x p_1^y & 0 & 0 & p_1^x p_1^y; \\ (1-p_1^x)p_2^y & 1-p_2^y & 0 & p_1^x p_2^y; \\ (1-p_1^y)p_2^x & 0 & 1-p_2^x & p_1^y p_2^x; \\ 0 & p_2^x(1-p_2^y) & p_2^y(1-p_2^x) & p_{44} \end{array} \right)$$

where $p_{44} = p_2^x p_2^y + (1-p_2^x)(1-p_2^y)$.

If $p_1^x, p_1^y, p_2^x, p_2^y > 0$, the Markov Model is positive-recurrent. The steady state distribution is the following:

$$\vec{\pi} = \left( \begin{array}{c} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \end{array} \right) = \frac{1}{k_1+k_2+k_3+k_4} \left( \begin{array}{c} k_1 \\ k_2 \\ k_3 \\ k_4 \end{array} \right)$$

where

$$\vec{k} = \left( \begin{array}{c} p_2^x p_2^y[(1-p_1^x)p_2^x(1-p_2^y) + (1-p_2^x)p_2^y(1-p_1^y)] \\ p_1^x p_1^y (p_2^x)^2(1-p_2^y) \\ p_1^x p_1^y (p_2^y)^2(1-p_2^x) \\ p_1^x p_1^y p_2^x p_2^y \end{array} \right) \tag{1}$$

The corresponding *throughput* and *cost* of node $x$ are:

$$\begin{aligned} T_x \quad = \quad & \pi_1(p_1^x)(1-p_1^y) + \pi_2(p_1^x)(1-p_2^y) + \\ & \pi_3(p_2^x)(1-p_1^y) + \pi_4(p_2^x)(1-p_2^y). \end{aligned} \tag{2}$$

$$C_x \quad = \quad \pi_1(p_1^x) + \pi_2(p_1^x) + \pi_3(p_2^x) + \pi_4(p_2^x). \tag{3}$$

Nodes may have physical limitations (e.g. power consumption constraints or application throughput constraints) that may bound its cost function. We bound allowed cost by a *budget*, $B_x$, such that a node's parameters must produce a cost $C_x \le B_x$.

When we consider cooperative nodes that seek to maximize throughput, we are also interested in system *fairness*: all nodes should get an equal share of the aggregate throughput. In addition, we assume that it is undesirable for any one node to "capture" the medium for an extended number of slots - a long-term capture can be thought of as unfair over a short duration. Koksal's work [28] gives an analysis of the short-term fairness of MAC protocols. It provides some insight into why MAC protocols exhibit bad short-term fairness using two different fairness indexes. In this paper, we measure short-term fairness via a more fundamental quantity defined as the following:

**Definition** *1:* Let $D_x$ be the number of consecutive slots following an initially successful transmission over which node $x$ successfully transmits packets (i.e., if there are $k$ successful consecutive transmissions, then $D_x = k-1$). The system is said to be ***M*-short-term fair** to all nodes if $E[D_x] \le M$ for all nodes $x$. ∎

**Remark:** By definition, if a system is $M$-short-term fair, it is also $N$-short-term fair for any $N > M$. The short-term fairness requires that each node cannot keep using the channel for an excessive amount of time when it "captures" the channel. This fairness guarantees the fair share of the channel when the time scale is small. Moreover, under short-term fairness, nodes will not be delayed for a very long time until a new successful transmission.

## IV. Cooperative Performance Analysis

In this section, we assume that nodes cooperate to fairly (i.e., equally) share the available bandwidth and to maximize the aggregate system throughput. By doing so, each node achieves the maximum throughput possible in a fair allocation when limited to protocols that cannot sense the wireless medium. Clearly, if it were permissible to bias the allocation toward one of the nodes, the system could achieve full utilization by allowing only one node to transmit at all time. If a centralized scheduler or carrier sensing mechanism were permitted, we could also make fair share of the medium with almost $100\%$ utilization. Here, we seek an unbiased and distributed solution for all nodes such that nodes will achieve the same performance on average.

Our goal in this section is to answer the following questions:

1) What are the values of $p_1^x$ and $p_2^x$ for each node $x$ that maximize the total throughput of the system?
2) What is this maximum achievable throughput of the system?
3) What is the short-term fairness of the optimal allocation, and how can that short-term fairness be improved?

**Theorem** *1:* For two homogeneous nodes with $p_1^x = p_1^y = p_1$ and $p_2^x = p_2^y = p_2$, $\sup\{T_x + T_y\} = 2/3$.

**Proof:** Substitute all the transmitting probabilities with $p_1$ and $p_2$ into Equation (1) and (2), we have

$$\vec{k} = \begin{pmatrix} 2p_2(1-p_2)(1-p_1) \\ (1-p_2)p_1^2 \\ (1-p_2)p_1^2 \\ p_1^2 \end{pmatrix},$$

and

$$\begin{aligned} T_x &= \pi_1 p_1(1-p_1) + \pi_2 p_1(1-p_2) \\ &\quad +\pi_3 p_2(1-p_1) + \pi_4 p_2(1-p_2) \\ &= \beta p_1(p_1^2 - \alpha p_1 + \alpha)/(p_1^2 - \alpha\beta p_1 + \alpha\beta), \end{aligned}$$

where

$$\alpha = 2p_2, \ \beta = (1-p_2)/(3-2p_2).$$

When $p_1 = 1$, $T_x = \beta = (1-p_2)/(3-2p_2)$ and $\beta \to 1/3$ as $p_2 \to 0$. By symmetry, $T_x + T_y \to 2/3$ as $p_2 \to 0$.

Next, we want to show $T_x < 1/3$ for all $p_1, p_2 \in (0, 1]$. It is equivalent to show the following:

$$\begin{aligned} & \beta p_1(p_1^2 - \alpha p_1 + \alpha)/(p_1^2 - \alpha\beta p_1 + \alpha\beta) < 1/3 \\ \iff \quad & 3\beta p_1(p_1^2 - \alpha p_1 + \alpha) < p_1^2 - \alpha\beta p_1 + \alpha\beta \\ \iff \quad & 3\beta p_1^3 - (3\alpha\beta + 1)p_1^2 + 4\alpha\beta p_1 - \alpha\beta < 0. \end{aligned}$$

We define $f(p_1) = 3\beta p_1^3 - (3\alpha\beta+1)p_1^2 + 4\alpha\beta p_1 - \alpha\beta$. Two boundary conditions are $f(0) = -\alpha\beta < 0$ and $f(1) = 3\beta - 1 < 0$. Since $f(p_1)$ is a cubic function of $p_1$, it is sufficient to show that the local maximum is less than zero, so as to prove that for any $p_1 \in (0, 1]$, $f(p_1) < 0$. At the local maximum,

$$f^{'}(p_1^*) = 9\beta(p_1^*)^2 - 2(3\alpha\beta + 1)p_1^* + 4\alpha\beta = 0.$$

Using the above condition, it is equivalent to show

$$f(p_1^*) = -(1/3)(3\alpha\beta + 1)(p_1^*)^2 + (8/3)\alpha\beta p_1^* - \alpha\beta < 0.$$

The maximum of the above function is

$$[(4/3)(3\alpha\beta + 1)\alpha\beta - ((8/3)\alpha\beta)^2]/[-(4/3)(3\alpha\beta + 1)].$$

The denominator is negative, while the numerator is positive because

$$\begin{aligned} & (4/3)(3\alpha\beta + 1)\alpha\beta - ((8/3)\alpha\beta)^2 > 0 \\ \iff \quad & (4/3)(3\alpha\beta + 1) - (64/9)\alpha\beta > 0 \\ \iff \quad & \alpha\beta < 3/7 \\ \iff \quad & 2p_2(1-p_2)/(3-2p_2) < 3/7 \\ \iff \quad & 14p_2^2 - 20p_2 + 9 > 0. \end{aligned}$$

Finally, because the local maximum $f(p_1^*) < 0$, we conclude that $f(p_1) < 0$ for all $p_1 \in [0, 1)$. ∎

Theorem 1 upper-bounds the maximum fair throughput at 2/3, which is achieved in the limit as both nodes choose $\{p_1 = 1, p_2 \to 0\}$. This solution is intuitive: collisions are less likely to occur in a carrier-sense free environment when nodes are very unlikely to start transmiting, but hold the medium until a subsequent collision.

**Theorem** *2:* For $N$ homogeneous nodes with $p_1 = 1$, $p_2 \to 0$, the total throughput approaches $\frac{N}{2N-1}$.

**Proof:** Consider in each time-slot, the whole system is in certain state. We aggregate all the system states into the following two states. One state is the "Busy" state where only one of the nodes is transmitting in the time-slot. The other state is the "Idle or Collision" state where no node or more than one node are transmitting in the time-slot. The state transition diagram is shown in Figure 2.

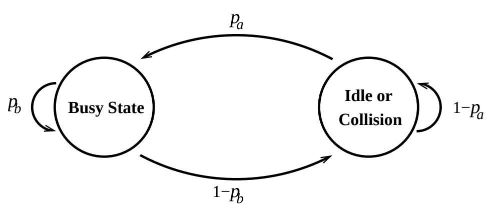



Fig. 2. N-node Markov Chain with $\{p_1 = 1, p_2 \to 0\}$.

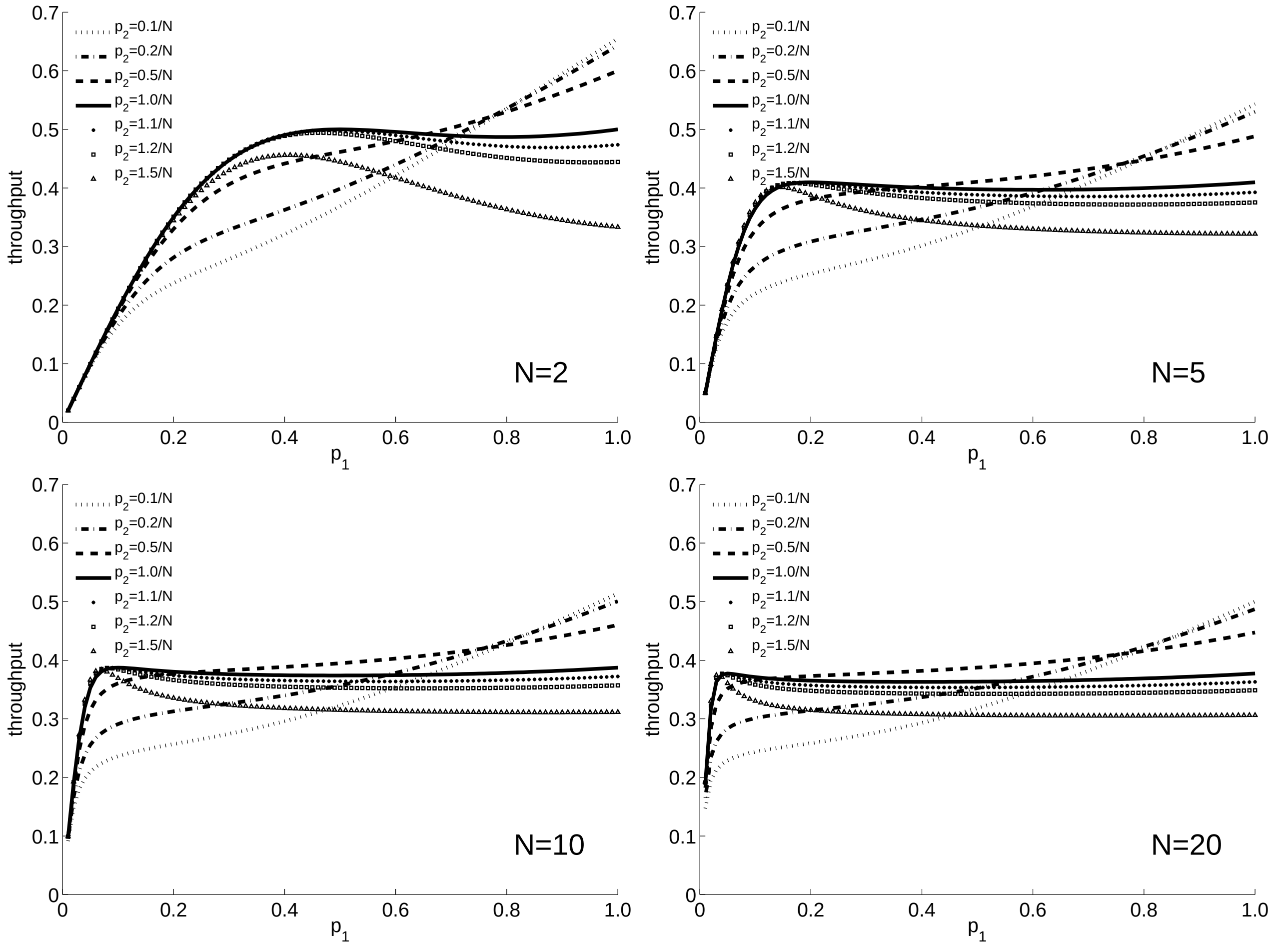



Fig. 3. Aggregate throughput for fixed $p_2$.

We define the transition probabilities as $p_b = (1-p_2)^{N-1}$ and $p_a = Np_2(1-p_2)^{N-1}$. $p_b$ indicates the probability that all of the $N-1$ backlogged nodes do not transmit. $p_a$ indicates the probability that only one of the $N$ nodes transmits. The system utilization becomes

$$\begin{aligned} \rho &= \pi_{busy} = p_a/(1-p_b+p_a) \\ &= \frac{Np_2(1-p_2)^{N-1}}{1-(1-p_2)^{N-1}+Np_2(1-p_2)^{N-1}} \\ &= N/(\frac{1}{p_2}\frac{1-(1-p_2)^{N-1}}{(1-p_2)^{N-1}}+N) \\ &= N/(\frac{1+(1-p_2)+(1-p_2)^2+\cdots+(1-p_2)^{N-2}}{(1-p_2)^{N-1}}+N). \end{aligned}$$

Therefore, $\rho \to \frac{N}{2N-1}$ as $p_2 \to 0$. ∎

Intuitively, when the number of nodes increases in the system, with higher probability, the channel is jammed with more than one node transmitting at the same time. Consequently, the aggregate system throughput decreases. However, Theorem 2 shows that the throughput does not drop to zero: even when the number of nodes tends to infinity, the aggregate throughput remains larger than one half. Note that this result differs from the traditional performance bound ($1/e$) of slotted-Aloha because our generalized model permits the capture of the resource. This allows a node to use the channel for very long but bounded intervals (given a fixed and non-zero value of $p_2$) of slots while all other nodes back off. An alternative analysis of this capture phenomenon can be found in [7].

Both Theorem 1 and Theorem 2 focus on the class of solutions where $p_1 = 1$ and $p_2 \to 0$. For $N = 2$, we proved in Theorem 1 that the optimal throughput is achieved at $\{p_1 = 1, p_2 \to 0\}$. Nevertheless, the maximum throughput is also achieved at $\{p_1 = 1, p_2 \to 0\}$ for $N > 2$. We present some evidence which shows the optimality of $\{p_1 = 1, p_2 \to 0\}$. The formal proof can be found in Appendix. We start with the following observation.

**Theorem** *3:* For $N$ homogeneous nodes, the solutions $\{p_1 = p_2 = 1/N\}$ and $\{p_1 = 1, p_2 = 1/N\}$ both achieve the throughput $(1-1/N)^{N-1}$.

**Proof:** When $\{p_1 = p_2 = 1/N\}$, each node tries to transmit at each time-slot independently with probability $1/N$. Therefore,

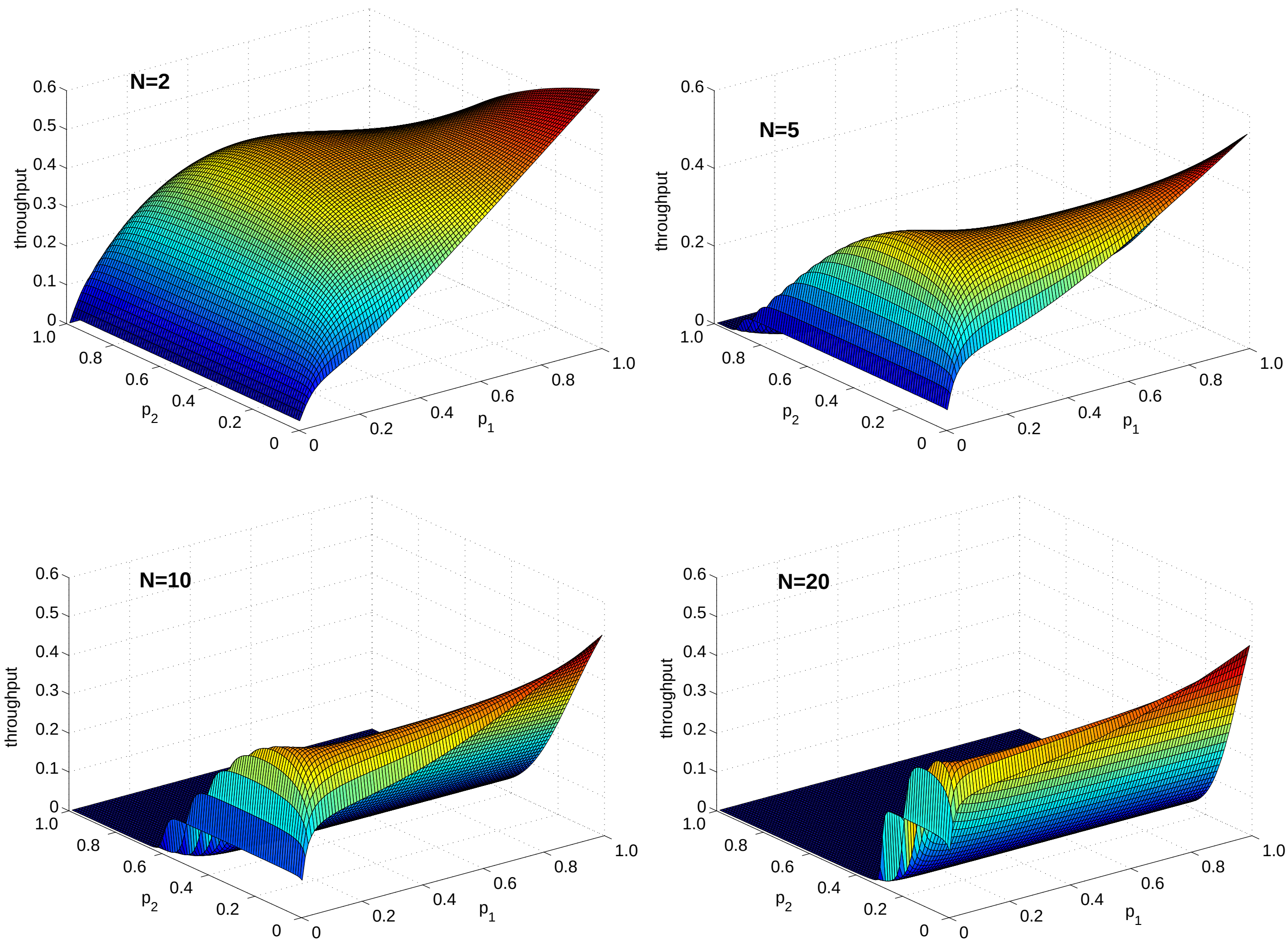



Fig. 4. Aggregate throughput.

the throughput is just the probability that if and only if one of the $N$ nodes transmits:

$$\rho = \binom{N}{1}(1/N)(1-1/N)^{N-1} = (1-1/N)^{N-1}.$$

When $p_1 = 1$ and $p_2 = 1/N$, we can adopt the Markov Chain in figure 2. We have the transition probabilities: $p_b = (1-p_2)^{N-1} = (1-1/N)^{N-1}$ and $p_a = Np_2(1-p_2)^{N-1} = (1-1/N)^{N-1}$. The throughput is

$$\rho = \pi_{busy} = p_a/(1-p_b+p_a) = p_a = (1-1/N)^{N-1}. \blacksquare$$

**Theorem 3 provides a reference point to divide the solution space into groups.** Figure 3 plots the throughput for systems of $N = 2, 5, 10$ and 20. In each subplot, $p_1$ varies along the $x$-axis, and different curves plot different values for $p_2$. For any $N$, we use the curve $p_2 = 1/N$ ( from Theorem 3, we know the exact value when $p_1 = 1$ or $p_1 = 1/N$) as a reference to divide solutions into two groups: $p_2 < 1/N$ and $p_2 > 1/N$. We plot the curve $p_2 = 1/N$ in solid lines. We compare the solution within each group and across groups, and make the following observations:

- When the value of $p_2$ increases from $1/N$, the throughput decreases. The curve of $p_2 = 1/N$ is above all curves of $p_2 > 1/N$.
- When the value of $p_2$ decreases from $1/N$, the throughput curve becomes lower in the region $p_1 \in (0, 1/N)$. But the maximum of each curve is at $p_1 = 1$. This maximum increases when $p_2$ decreases.

In Figure 4, we plot the surface of throughput for systems with various number of nodes. In each subplot, $p_1$ varies along the $x$-axis (on the right); $p_2$ varies along the $y$-axis (on the left). By increasing of the number of nodes $N$ in system, the surfaces bend down dramatically in the region of small $p_1$ and large $p_2$ (left corner). The high throughput is reached in the region of large $p_1$ and small $p_2$ (right corner). In particular, the maximum is achieved at $p_1 = 1, p_2 \to 0$.

Although the solution $\{p_1 = 1, p_2 \to 0\}$ might maximize throughput, it is not short-term fair, in which a single transmitter gains exclusive access of the medium for a long time. As $p_2 \to 0$, we have $E[D_x] \to \infty$. Afterward we consider how to enforce short-term fairness:

**Theorem** *4:* For $N$ homogeneous nodes with $p_1 = 1$ and $p_2 \geq 1 - \sqrt[N-1]{1 - 1/M}$, the system is $M$-short-term fair[3] to all nodes.

**Proof:** Because $D_x$ is a geometric random variable with parameter $1 - p_b$, we have

$$E[D_x] = \frac{1}{1 - (1 - p_2)^{N-1}}.$$

Since $p_2 \geq 1 - \sqrt[N-1]{1 - 1/M}$, we have $E[D_x] \leq M$. By definition, the system is $M$-short-term fair to all nodes. ■

Theorem 4 quantifies how to select $p_2$ to achieve a certain short-term fairness. In particular, in order to achieve $M$-short-term fairness, we can choose the following value of $p_2$:

$$p_2 = 1 - \sqrt[N-1]{1 - 1/M}. \tag{4}$$

The total throughput becomes a function of $M$:

$$\rho = \frac{Np_2(1 - p_2)^{N-1}}{1 + (Np_2 - 1)(1 - p_2)^{N-1}} \tag{5}$$

$$\Longrightarrow \rho = \frac{N(M-1)}{N(M-1) + \frac{1}{1 - \sqrt[N-1]{1 - \frac{1}{M}}}}. \tag{6}$$

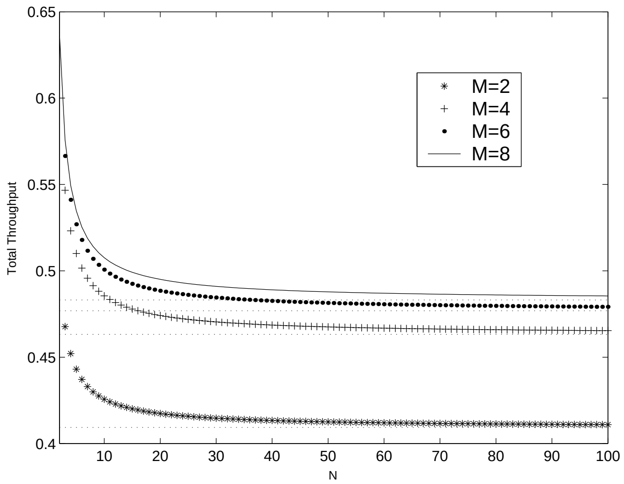



Fig. 5. Throughput under different fairness conditions.

Figure 5 plots the total throughput under different short-term fairness constraints $(M)$ as the number of nodes, $N$, varies along the $x$-axis. This figure shows the tradeoff between short-term fairness and throughput. However, without sacrificing much throughput, the system achieves desirable short-term fairness. For example, if we want the system to be 8-short-term fair, which means each node can successfully transmit no more than 8 consecutive slots on average when it captures the channel, we can achieve a total throughput close to $1/2$ even for large $N$. In fact, when $N \to \infty$, the total throughput does not collapse to zero. We will discuss the throughput limits in the next theorem.

**Lemma** *1:* For any constant $M > 0$, if $p_2 = 1 - \sqrt[N-1]{1 - 1/M}$, then $Np_2$ is monotonically decreasing with $N$.

**Proof:** Let $\alpha = 1 - 1/M$ and $\beta = \alpha^{\frac{1}{N-1}}$. We have

$$p_2 = 1 - \sqrt[N-1]{1 - 1/M} = 1 - \alpha^{\frac{1}{N-1}} = 1 - \beta.$$

First, $p_2$ is strictly decreasing in $N$, because $dp_2/dN = \ln \alpha \frac{\beta}{(N-1)^2} < 0$. We define $f(N) = (N-1)p_2$.

$$\begin{aligned} df(N)/dN &= (N-1)[\ln \alpha \tfrac{\beta}{(N-1)^2}] + 1 - \beta \\ &= ((N-1)^{-1} \ln \alpha - 1)\beta + 1 \\ &= (\ln \beta - 1)\beta + 1 \\ &= \beta \ln \beta - \beta + 1 < \ln \beta - \beta + 1 < 0. \end{aligned}$$

[3]see Definition 1

The last inequality holds because of the following. We define $g(\beta) = \ln\beta - \beta + 1$. $g(\beta)$ is a strictly concave function, because $g'(\beta) = 1/\beta - 1$ and $g''(\beta) = -1/\beta^2$. Since $g(1) = 0, g'(1) = 0$ and $g''(1) = -1 < 0$, we see $g(\beta)$ attains its maximum value 0 at $\beta = 1$. $\beta < 1$ holds under our context; therefore, the last inequality holds.

Finally, $Np_2 = (N-1)p_2 + p_2$, and consequently, $dNp_2/dN = df(N)/dN + dp_2/dN < 0$. ∎

**Theorem** *5:* Under any short-term fairness condition $E[D_x] = M$, the total throughput, $\rho$, is lower-bounded by $\frac{-(M-1)\ln(1-\frac{1}{M})}{1-(M-1)\ln(1-\frac{1}{M})}$.

**Proof:** We choose the value of $p_2$ in Equation (4) to satisfy the short-term fairness condition. Accordingly, from Equation (6), we have

$$\frac{1}{M-1}\frac{1}{\frac{1}{\rho}-1} = \left(1 - \sqrt[N-1]{1-\frac{1}{M}}\right)N = \frac{1-\sqrt[N-1]{1-\frac{1}{M}}}{\frac{1}{N}}.$$

The right hand side is in the form of $\frac{0}{0}$ as $N \to \infty$. By L'hospital's rule,

$$\lim_{N\to\infty}\frac{\ln(1-\frac{1}{M})\sqrt[N-1]{1-\frac{1}{M}}(N-1)^{-2}}{-N^{-2}} = -\ln\left(1-\frac{1}{M}\right)$$

$$\therefore \lim_{N\to\infty}\frac{1}{M-1}\frac{1}{\frac{1}{\rho}-1} = -\ln\left(1-\frac{1}{M}\right)$$

$$\Rightarrow \lim_{N\to\infty}\rho = \frac{-(M-1)\ln(1-\frac{1}{M})}{1-(M-1)\ln(1-\frac{1}{M})}.$$

On the other hand,

$$\frac{1}{M-1}\frac{1}{\frac{1}{\rho}-1} = (1-\sqrt[N-1]{1-\frac{1}{M}})N = Np_2.$$

Thus, by Lemma 1, $\rho$ is monotonically decreasing in $N$ (as well as in $Np_2$). Therefore, $\rho$ is lower-bounded by $\frac{-(M-1)\ln(1-\frac{1}{M})}{1-(M-1)\ln(1-\frac{1}{M})}$. ∎

Theorem 5 provides the lower bounds for the curves in Figure 5. We draw the limits of the throughput in dotted lines under each throughput curve. We see, for $M$ to be reasonably large (e.g. $M = 8$), the throughput lower limit is close to $1/2$. Indeed, when $M$ tends to infinity, this limit also tends to $1/2$. In practice, a fairness requirement $M$ can be achieved by choosing a suitable $p_2$ for all users. Like the information of the number of users (N), we suggest that the $p_2$ value can be broadcasted in the beacon messages by the system infrastructure.

## V. Competitive Performance Analysis

In the previous section, we identified the lower bounds of the obtainable throughput among cooperative nodes, even taking into account short-term fairness constraints. In this section, we assume that each node is autonomous and sets its protocol parameters to strategically maximize its own throughput, subject to currently observed conditions. First, let us see how a single node can increase its own throughput by deviating from the cooperative solution. After that, we construct a *Stackelberg game* [6] by formulating a constrained optimization problem for each node, maximizing its own throughput. The game solution reveals that a *Prisoner's Dilemma* [6] phenomenon can occur.

### *A. Selfish Behavior in a Cooperative Environment*

Suppose $N$ nodes are originally cooperative and use $p_1 = 1$ and $p_2 = 1 - \sqrt[N-1]{1-1/M}$ to achieve the maximum $M$-short-term fair aggregate throughput. In this system, each node $x$ obtains throughput:

$$T_x = \rho/N = (M-1)/[N(M-1) + 1/p_2].$$

If one node deviates from this cooperative solution and sets $p_2 = 1$ instead, its throughput increases to

$$T'_x = p_b = (1-p_2)^{N-1} = 1 - 1/M.$$

Its throughput now equals the probability that no other node is transmitting in each time-slot. Comparing the above two equalities, we have:

$$\frac{T'_x}{T_x} = \frac{N(M-1)+1/p_2}{M} = N + \frac{1-Np_2}{Mp_2}.$$

Hence, by unilaterally changing $p_2$ to be 1, a selfish node can usually increase its throughput at least $N$ times (if $Np_2 < 1$). This change sacrifices the throughput of all other nodes, which no longer obtain any throughput. In fact, given all other nodes' parameters fixed, any node can unilaterally increase its parameters to obtain higher throughput. This also implies that the Nash equilibrium is where all nodes behave aggressively and greedily, i.e., $p_1 = p_2 = 1$, but the aggregate throughput equals zero.

### B. Stackelberg Game

We now explore what happens when multiple nodes set their parameters in a greedy manner. As shown in the previous subsection, the Nash equilibrium is inefficient and undesirable for all users. Instead of exploring the inefficient Nash equilibria directly, we start with a *Stackelberg game*[6], which enables one of the nodes to fix its optimal parameters, and then let other nodes respond accordingly. Stackelberg games have been applied to different areas of networking protocols (e.g. routing strategies [29], [30]) in order to achieve efficient equilibria. Later, we will extend our model to simultaneous move games from which the Prisoner's Dilemma phenomenon induces the inefficiency of aggregate throughput in the system.

Here, we consider a network that consists only of two selfish nodes $x$ and $y$, each of which wants to maximize its own throughput. In addition, we assume that each has budget constraints $C_x \leq B_x$ and $C_y \leq B_y$ respectively. $C_x$ and $C_y$ are the costs of both nodes as defined in Equation (3). $B_x, B_y \in (0, 1]$ are two budget constants that physically restrict the average number of packets the node can transmit in each time-slot. We impose these budget constraints in order to model the nodes in a wireless ad-hoc network or a sensor network. Because nodes in these networks are very sensitive to power consumption, and transmitting packets consumes a lot of battery power. Consequently, the behavior of nodes may largely depend on their budget constraints.

In the Stackelberg competition between these two nodes, a "leader" chooses a strategy (i.e. the transmitting probabilities in both the Free State and the Backlogged State), and then a "follower", informed of the leader's choice, chooses a counter-strategy. We formulate the non-cooperative Stackelberg game as follows:

| | |
|---|---|
| Players: | The leader node $x$ and the follower node $y$. |
| Strategy: | $S^x = \{p_1^x, p_2^x\}$ for $x$; $S^y = \{p_1^y, p_2^y\}$ for $y$. |
| Payoff: | $T_x$ and $T_y$ for $x$ and $y$ respectively. |
| Game rule: | $x$ decides $\{p_1^x, p_2^x\}$ first. $y$ then decides $\{p_1^y, p_2^y\}$ after knowing $\{p_1^x, p_2^x\}$. |

**Follower's Problem:**

The follower $y$ is given the leader's chosen parameters. It then simply sets its own parameters to maximize its own throughput. More formally, for any given $\widetilde{S^x}$, the follower node $y$ solves:

$$\hat{S^y}(\widetilde{S^x}) = \arg\max \; T_y(\widetilde{S^x}, \hat{S^y})$$
$$\texttt{Subject to}: C_y(\widetilde{S^x}, \hat{S^y}) \leq B_y.$$

**Leader's Problem:**

The leader knows that the follower will choose its parameters to greedily maximize its own throughput. Therefore, the leader must choose its protocol parameters that will maximize its throughput, given the follower will subsequently choose its own parameters to maximize its throughput. More formally, the leader node $x$ solves:

$$\hat{S^x} = \arg\max \; T_x(\hat{S^x}, \hat{S^y}(\hat{S^x}))$$
$$\texttt{Subject to}: C_x(\hat{S^x}, \hat{S^y}(\hat{S^x})) \leq B_x.$$

In practice, users might not know the strategy of other users. The Stackelberg game assumes that the follower knows the strategy of the leader before deciding its own strategy. Both the leader and the follower are not necessarily very different; however, the leader is more strategically sophisticated and the follower is simply a throughput maximizer. We will relax this information structure to let both players decide their strategies simultaneously later.

In order to solve this Stackelberg game, we first solve the follower's problem for every possible strategy taken by node $x$. Thus, we obtain the best response strategy of $y$ as a function of node $x$'s strategy. After that, the leader decides its optimal strategy according to node $y$'s best response strategy. This procedure is often referred to as *backwards induction*[4] [17]. The corresponding game solution is often referred to as a Stackelberg equilibrium. We derive numerical results based on Equation (2) and (3), whereas close-form solutions for these optimization problems on Markov Chain might not be available.

### C. Three Stackelberg Equilibrium Regions

We solve the above Stackelberg game for nodes who have the same budget constraints, i.e., $B_x = B_y$.

Figure 6 and Figure 7 show the throughput and costs of both players in Stackelberg equilibrium. The x-axis represents the budget constraint for both players. The change in the throughput as a function of the budget behaves differently in three different solution regions:

1) When the budget is less than $1/3$, both players achieve the same throughput, and they both use up their budgets. Within this region, an increase of budget improves the achieved throughput. The throughput is mainly limited by the budget constraints rather than the competition between these two players.

[4]Backward induction is actually a more general procedure to identify the Subgame Perfect Nash Equilibria in any finite dynamic game with perfect information.

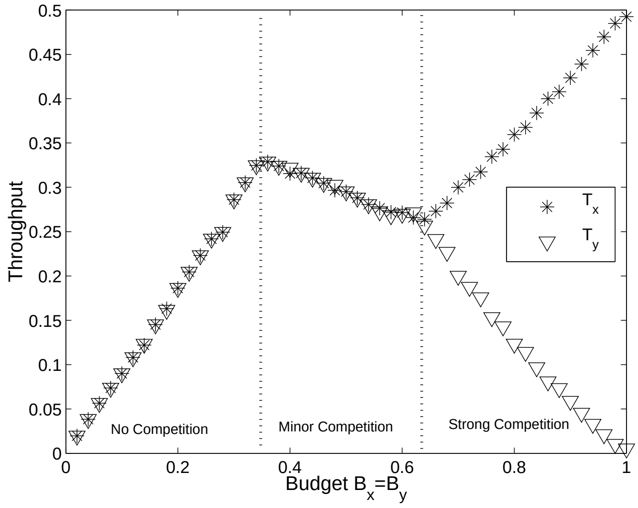



Fig. 6. Throughput in Stackelberg equilibrium.

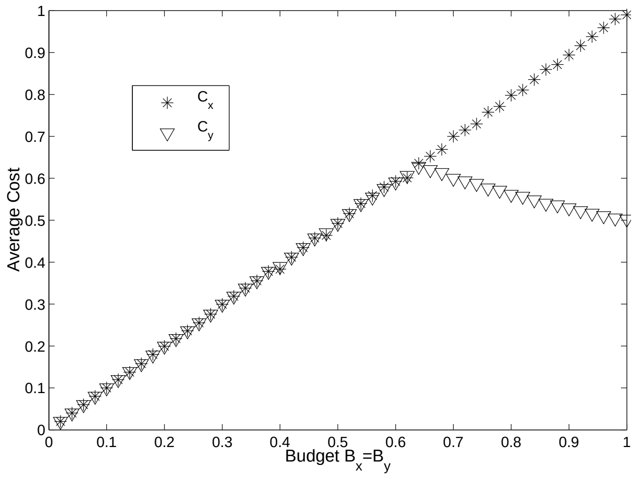


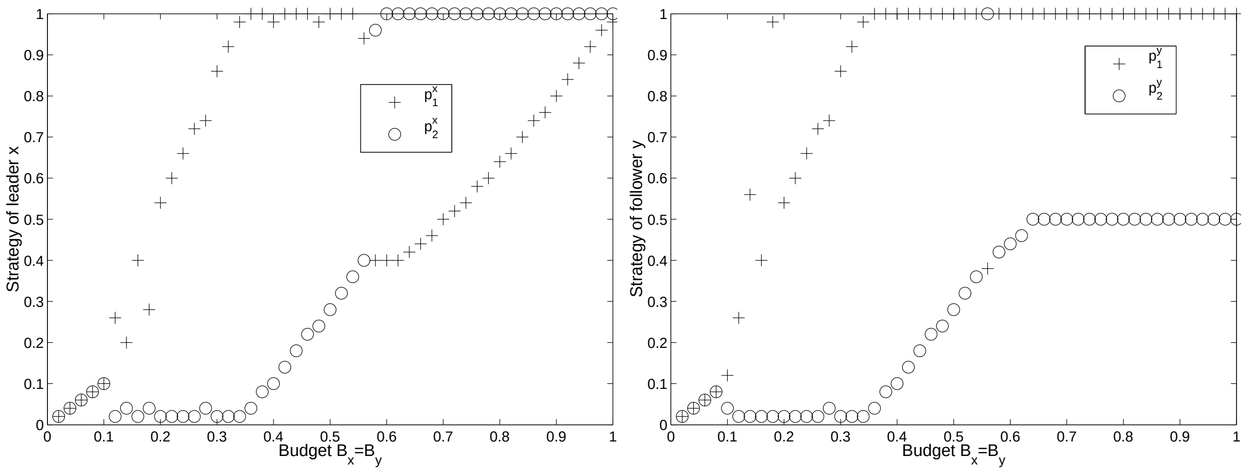

Fig. 7. Cost in Stackelberg equilibrium.

2) When the budget is between $1/3$ and $2/3$, both players again achieve similar throughput and use up their budgets. However, an increase of budget deteriorates each player's throughput. In this region, not only does the budget constraints limit the throughput, but the competition between these two players further reduces the throughput as well.
3) When the budget is more than $2/3$, the leader can select parameters that give it a larger fraction of the throughput. As the budget increases, this unfair allocation of throughput exacerbates and the follower, still wishing to maximize its own throughput, actually becomes less aggressive and uses a partial budget. In this region, the Stackelberg game benefits the leader by sacrificing the throughput of the follower.

Figure 8 plots values of $p_1$ and $p_2$, revealing the strategies of both players in Stackelberg equilibrium. In the first two solution regions, both players use similar strategies. As a result, it does not matter (to a node) whether it is the leader or the follower, because both players achieve similar throughput. In particular, when the budgets are close to $1/3$, the strategies selected by the players are similar to what would be selected by cooperative players, and the aggregate throughput approaches $2/3$. As the budgets are further increased, the nodes' additional contention on the medium and the rate of interference become significant.

When the budgets exceed $2/3$, the leader's strategy changes dramatically. Instead of setting $p_1 = 1$, it sets $p_2 = 1$. This implies that if a transmission fails in a slot, it will attempt to retransmit during the next slot. This makes sense intuitively because the follower, attempting to maximize its own throughput with its confined budget, must back off with high probability after a collision, and the "safest" time for the follower to transmit will be following a previous successful transmission. Because the leader uses $p_2 = 1$, the follower can only successfully transmit when the leader is in the Free State. Therefore, the follower must not fully use its budget, since it has to reduce the collision probability that leads the leader to a Backlogged State.

Noticeably, when the leader starts to set $p_2 = 1$ around a budget of $0.55$, the follower's strategy also follows it at that budget, since the follower can afford to mimic the leader's strategy to maximize its own throughput at that point. When the




(a) Leader's Strategy

(b) Follower's Strategy

Fig. 8. Strategies in Stackelberg equilibrium.

budget in further increased, the follower cannot use the same aggressive strategy to maximize its own throughput.

### D. Prisoner's Dilemma

Being the leader player in the above Stackelberg game, node $x$ achieves higher throughput than it can gain in a cooperative environment. To relax this discrimination, we now assume that both nodes will decide their strategies simultaneously. This models situations in wireless network where users do not know other users' backoff rates, which can be any strategy of $p_1$ and $p_2$, in advance. We focus on three representative budget scenarios and the corresponding strategies (from Figure 8) that would be played in the Stackelberg game:

- Low budget region: $B_x = B_y = 0.34$. Strategy $S_C = S^x = S^y = \{p_1 = 0.98, p_2 = 0.02\}$.
- Medium budget region: $B_x = B_y = 0.5$. Strategy $S_M = S^x = S^y = \{p_1 = 1, p_2 = 0.28\}$.
- High budget region: $B_x = B_y = 0.8$. Strategy $S_L = S^x = \{p_1 = 0.64, p_2 = 1\}$, and $S_F = S^y = \{p_1 = 1, p_2 = 0.5\}$.

Strategy $S_C$ in the lower budget region is similar to the strategy played by the nodes in a cooperative environment. Strategy $S_M$, more aggressive than $S_C$, is played by both the leader and the follower in the medium budget region. Finally, $S_L$ and $S_F$ are the strategies of the leader and the follower in the high budget region.

We consider two simultaneous move games, where two nodes must choose their parameters to maximize their individual throughput without knowing what their opponent chooses to do. Each game models a budget scenario where nodes are confined to use some of representative strategies above.

**To Cooperate or to Compete?**

First, we consider a game in which two nodes, each with a budget of 0.5, must decide whether they will cooperate or behave in a greedy manner (i.e., should the node set its parameters according to $S_C$ or $S_M$?) By evaluating Equation (2) for both nodes, we show the throughput of them in the following table.

| | $S_C$ | $S_M$ |
|---|---|---|
| $S_C$ | (0.3246,0.3246) | (0.0034,0.9288) |
| $S_M$ | (0.9288,0.0034) | **(0.2951,0.2951)** |

The most efficient solution is at $(S_C, S_C)$. However, a selfish node will note that whichever strategy its opponent chooses, its throughput will be increased by choosing $S_M$. Here, we see a typical Prisoner's Dilemma [6]. Although from a global perspective, both players know the best solution is $(S_C, S_C)$, from any hypothetical local point, strategy $S_M$ should always be played. This is because, for any fixed strategy by the opponent, choosing $S_M$ is always better than choosing $S_C$. Strategy $S_M$ is called the *dominating strategy* [6] for both players and the solution $(S_M, S_M)$ is the unique *Nash equilibrium* [6] solution of this game.

**To Lead (be aggressive) or to Follow (be mild)?**

In the second game, we assume that nodes have budgets in the third region. As in the Stackelberg game, the leader player is better off by playing an aggressive strategy, however; now the nodes must also decide whether to choose the leader's strategy or the follower's strategy, without knowing the other player's response in advance. Notice that there is no actual leader or follower in the simultaneous move game. Here, nodes are restricted to the strategies used by the leader and the follower in the Stackelberg game.

| | $S_F$ | $S_L$ |
|---|---|---|
| $S_F$ | (0.25,0.25) | **(0.1233,0.3595)** |
| $S_L$ | **(0.3595,0.1233)** | (0,0) |

Here, a node's best strategy is not clear. A node is always better off choosing the opposing strategy of its competitor. Choosing the follower strategy is more conservative. A throughput of at least $0.1233$ is ensured, but the throughput can be at most $0.25$. If the leader strategy is chosen, throughput of $0.3595$ is possible, but throughput of $0$ is also a possible outcome. Interestingly, this game has two symmetric Nash equilibrium solutions, i.e., $(S_F, S_L)$ and $(S_L, S_F)$.

From the results of the above two simultaneous move games, we can further explain the three solution regions of the Stackelberg game in figure 6. When the budget is between $1/3$ and $2/3$, both nodes are afford to use classes of cooperative (e.g. $S_C$) or competitive (e.g. $S_M$) strategies. The uniqueness of the Nash equilibrium solution implies that nodes would similar strategies regardless it is the leader player or the follow player. As the budget increases, it is affordable for the nodes to use any strategy. From the symmetric Nash equilibra in the simultaneous move game, the leader player can always take advantage to choose a favorable equilibrium in the Stackelberg game.

## VI. Selfish Behavior Detection and Prevention

In the previous section, we used non-cooperative games of two nodes to show that selfish behavior of nodes deteriorates the overall throughput obtained across the transmission medium, as well as that of the individual nodes. In this section, we discuss how cooperative nodes can identify and prevent selfish behavior in a general $N$-node system.

### A. Transmitting is a Dominating Strategy

Consider any node $i$ at any time-slot $t$. If it attempts to transmit, the probability of success is

$$\Pi_{j\neq i}(1-p^j),$$

where $p^j$ is the probability ($p_1^j$ or $p_2^j$ depending on $j$'s state) that node $j$ transmits in that time-slot. Without any budget constraint, node $i$ can achieve the highest throughput by transmitting a packet during every time-slot. However, if node $i$ transmits a packet in every time-slot, other nodes transmission attempts will always fail. Over time, this phenomenon can be easily observed. Here, we consider how cooperative nodes can alter their parameters if their perceived *successful rates* are too small in such a way that selfish nodes become "encouraged" to set their parameters to mimic the behavior of cooperative nodes.

### B. Selfish Behavior Detection

**Theorem** *6:* For an $M$-short-term fair cooperative environment, where each node uses $p_1 = 1$ and $p_2 = 1 - \sqrt[N-1]{1-1/M}$, the *successful rate* defined by $T_x/C_x$ for any node $x$ is lower-bounded by $\frac{(M-1)(M-2)}{(M-1)(M-2)+1}$.

**Proof:**

$$\frac{T_x}{C_x} = \frac{NT_x}{NC_x} = \frac{\rho}{NC_x},$$

where $NC_x$ is equal to the total average cost for all nodes. Suppose all $N$ nodes are in backlogged state. Let Q to be the number of nodes that decide to transmit in a time-slot. Therefore, $Pr\{Q=i\} = q_i = \binom{N}{i} p_2^i (1-p_2)^{N-i}$, where $Q$ is a binomial random variable with parameter $p_2$ and $N$.

$$\begin{aligned} NC_x &= \rho + (1-\rho)\textstyle\sum_{j=2}^{N} jq_j/(1-q_1) \\ &= \rho + (1-\rho)(E[Q]-q_1)/(1-q_1) \\ &= \rho + (1-\rho)(Np_2-q_1)/(1-q_1). \end{aligned}$$

Since $q_1 = Np_2(1-p_2)^{N-1} = Np_2(1-\frac{1}{M})$ and $\frac{1}{\rho} - 1 = \frac{1}{M-1}\frac{1}{Np_2}$, we obtain

$$\begin{aligned} & T_x/C_x = \rho/[\rho + (1-\rho)(Np_2-q_1)/(1-q_1)] \\ \Longleftrightarrow\quad & \tfrac{1}{T_x/C_x} - 1 = (\tfrac{1}{\rho}-1)(Np_2-q_1)/(1-q_1) \\ \Longleftrightarrow\quad & \tfrac{1}{T_x/C_x} - 1 = \tfrac{1}{M-1}\tfrac{1}{Np_2}(Np_2 - Np_2(1-\tfrac{1}{M}))/(1-q_1) \\ \Longleftrightarrow\quad & \tfrac{1}{T_x/C_x} - 1 = \tfrac{1}{M-1}(\tfrac{1}{M})/(1-q_1) \\ \Longleftrightarrow\quad & \tfrac{1}{T_x/C_x} - 1 = \tfrac{1}{M-1}(\tfrac{1}{M})/(1-Np_2(1-\tfrac{1}{M})). \end{aligned}$$

By Lemma 1, $Np_2$ is monotonically decreasing in $N$. When $N=2$, $Np_2 = 2/M$ is the maximum for $N>1$. Substituting $Np_2$ with $2/M$, we have

$$\begin{aligned} & \tfrac{1}{T_x/C_x} - 1 = \tfrac{1}{M-1}(\tfrac{1}{M})/(1-Np_2(1-\tfrac{1}{M})) \\ \Longrightarrow\quad & \tfrac{1}{T_x/C_x} - 1 \leq \tfrac{1}{M-1}(\tfrac{1}{M})/(1-\tfrac{2}{M}(1-\tfrac{1}{M})) \\ \Longrightarrow\quad & \tfrac{1}{T_x/C_x} - 1 \leq \tfrac{1}{M-1}/(M-2(1-\tfrac{1}{M})) < \tfrac{1}{M-1}\tfrac{1}{M-2} \\ \Longrightarrow\quad & T_x/C_x > [(M-1)(M-2)]/[(M-1)(M-2)+1]. \end{aligned}$$

■

Theorem 6 provides a guideline for cooperative nodes to detect the existence of any selfish node distributively. A fraction of at least $\frac{(M-1)(M-2)}{(M-1)(M-2)+1}$ of a cooperative node's transmissions should be successful. For instance, when $M$ equals 8, this average success rate lower-bound is $\frac{42}{43}$. When $M$ is larger, the success rate is even higher. Notice that this success rate is different from the throughput that a node achieves. With the increase of the number of nodes, each node's throughput decreases however the success rate lower-bound remains the same. In practice, each node can measure this quantity to infer if there is any selfish node in the system.

### *C. Selfish Behavior Prevention*

In order to prevent selfish behavior in the system, each cooperative node could implement a new strategy when they detect selfish nodes. The new strategy uses a $p_2'(> p_2)$ that reduces the throughput of the selfish node to a level below what it would have achieved if $p_2$ were used by all cooperative nodes. Knowing that such a reduction will occur, selfish nodes have the necessary incentive to remain cooperative.

Suppose after all cooperative nodes activate the new strategy $p_2'$, the selfish node obtains throughput $T_x'$. $T_x'$ has to be less than $\rho/N$, which is the fair share throughput gained in a cooperative environment:

$$\begin{aligned} & T_x' = (1-p_2')^{N-1} < \rho/N \\ \Longleftrightarrow \quad & 1-p_2' < \sqrt[N-1]{\rho/N} \\ \Longleftrightarrow \quad & p_2' > 1 - \sqrt[N-1]{\rho/N}. \end{aligned}$$

From Theorem 2, we know that $\rho$ is lower-bounded by $1/2$. Hence, we can substitute in $1/2$ for $\rho$ when calculating the lower-bound of $p_2'$ as an approximation.

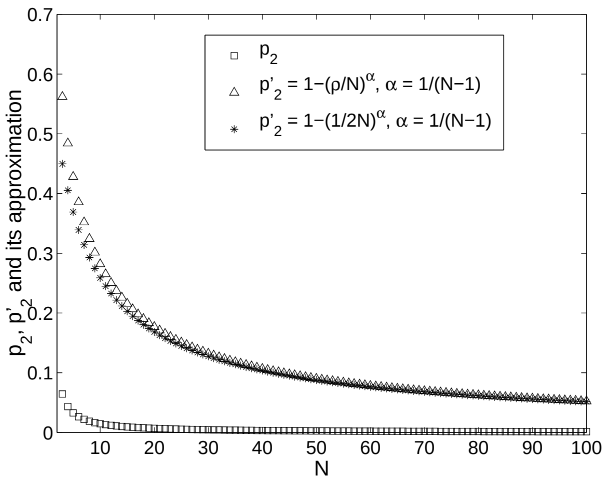



Fig. 9. Cooperative $p_2$, selfish preventive $p_2'$ and its approximation.

Figure 9 shows the cooperative strategy $p_2$ and the selfish prevention strategy $p_2'$. We see $1 - \sqrt[N-1]{1/2N}$ is a good approximation for the lower-bound for $p_2'$.

In practice, the counter-strategy $p_2'$ can be considered as a Tic-for-Tac incentive mechanism to encourage the cooperation of all users. To implement the mechanism, each node broadcasts the detection of selfish node and starts to use $p_2'$ for a fixed amount of slots. Other cooperative nodes will also activate the strategy $p_2'$ for the same amount of time. Within this "lockup" period, system throughput is reduced as well as the selfish users. After the lockup period, all the nodes reset their strategy to be cooperative and bring the system back to normal.

## VII. **Adversarial Model Analysis**

All previous scenarios assume that each node, whether cooperative or selfish, is interested in maximizing its own throughput. In this section, we consider an attacking node whose goal is to use its restricted budget to minimize the throughput of the other nodes in the system, i.e., to cause as many of its packets to collide with what would otherwise be successful transmissions. We first discuss how much damage it will cause if an adversary node uses a random (stateless) attack. Next, we formulate this attack model as another Stackelberg game.

### A. Pure Random (Stateless) Attack

If an attacking node is able to transmit a packet in every time-slot, it can clearly jam all transmissions. We assume that the adversary node has a budget $B \in (0, 1]$, allowing it to transmit in at most a fraction $B$ of the slots. This budget can be considered as the highest frequency of transmission under which an attack cannot be detected.

**Definition** *2:* An adversary node uses ***p*-pure random attack** if it transmits a packet in each time-slot independently with probability $p$. ∎

By Definition 2, an adversary node with a budget $B$ can use a $p$-pure random attack for any $p \leq B$. We can imagine that $p$-pure random attack for a communication channel is identical to a lossy channel where a packet is lost with probability $p$.

**Theorem** *7:* Suppose there are two nodes $x$ and $y$ in the system. If node $x$ is an adversary node that uses $p$-pure random attack, then regardless of the strategy of player $y$, its throughput $T_y$ is equal to $(1-p)C_y$.

**Proof:** Substitute $p_1^x$ and $p_2^x$ with $p$ in the corresponding throughput for $y$ as in Equation (2). We have

$$\begin{aligned} T_y &= (\pi_1, \pi_3, \pi_2, \pi_4) \begin{pmatrix} p_1^y(1-p_1^x) \\ p_1^y(1-p_2^x) \\ p_2^y(1-p_1^x) \\ p_2^y(1-p_2^x) \end{pmatrix} \\ &= (1-p)(\pi_1, \pi_3, \pi_2, \pi_4) \begin{pmatrix} p_1^y \\ p_1^y \\ p_2^y \\ p_2^y \end{pmatrix}. \end{aligned}$$

Since the corresponding cost function for $y$ as in Equation (3), we have

$$C_y = (\pi_1, \pi_3, \pi_2, \pi_4) \begin{pmatrix} p_1^y \\ p_1^y \\ p_2^y \\ p_2^y \end{pmatrix}.$$

Therefore, $T_y = (1-p)C_y$. ∎

Theorem 7 formalizes the intuitive result that a $p$-pure random attack reduces the capacity to be $1-p$ of the original capacity. Interestingly and counter-intuitively, if we have more than one cooperative node, the damage caused by a $p$-pure random attack is often larger than a factor of $1-p$.

**Theorem** *8:* Suppose originally there are $N$ homogeneous nodes that use $p_1 = 1$ and $p_2 < 1/N$ in the system. They achieve an aggregate throughput $\rho$. If an adversary node joins the system and uses $p$-pure random attack, then the aggregate throughput of the $N$ cooperative node is less than $(1-p)\rho$.

**Proof:** Before the adversary node comes into the system, we can model the system as in Figure 2. The transition probabilities are $p_b = (1-p_2)^{N-1}$ and $p_a = Np_2(1-p_2)^{N-1}$. After the adversary node comes, we define the corresponding transition probabilities to be $p_b'$ and $p_a'$. Because a successful packet from a normal node happens only if the adversary node does not transmit, we have $p_b' = (1-p)p_b$ and $p_a' = (1-p)p_a$. From Equation (5), we obtain

$$\rho = \frac{Np_2(1-p_2)^{N-1}}{1+(Np_2-1)(1-p_2)^{N-1}}.$$

The new throughput $\rho'$ is

$$\rho' = \frac{p_a'}{1-p_b'+p_a'} = \frac{Np_2(1-p_2)^{N-1}}{\frac{1}{1-p}+(Np_2-1)(1-p_2)^{N-1}}.$$

Therefore,

$$\frac{\rho'}{\rho} = \frac{1+(Np_2-1)(1-p_2)^{N-1}}{\frac{1}{1-p}+(Np_2-1)(1-p_2)^{N-1}} < 1-p.$$

The last inequality holds if $p_2 < 1/N$. ∎

An explanation of this result is as follows. As more nodes participate in the cooperative process, the expected number of slots between transmissions in the Backlogged State grows at a faster rate than the expected number of slots between transmissions in the Free State. A random seeding of losses forces more nodes to spend more time in the Backlogged State, and as a result, each node attempts fewer transmissions over time, yet still loses a fraction $p$ of the attempts to the random loss process.

### *B. Adversary Stackelberg Game*

Now, we compute the reduction in throughput that an adversary node can cause if it maximizes its attack power under a two-state system. Like in section V-B, we model this system as a Stackelberg game in this section. The difference between the previous model and this model is that we assume the leader node $x$ is the attacker and its sole objective is to minimize the throughput of node $y$. Because the leader has an advantage over the follower, making the adversary node the leader maximizes its potential for damage. We still assume that node $x$ and $y$ have budget constraints: $C_x \leq B_x$ and $C_y \leq B_y$ respectively. The adversary Stackelberg game can be formally described as follows:

| | |
|---|---|
| Player: | The leader node $x$ and the follower node $y$. |
| Strategy: | $S^x = \{p_1^x, p_2^x\}$ for $x$; $S^y = \{p_1^y, p_2^y\}$ for $y$. |
| Payoff: | $-T_y$ and $T_y$ for $x$ and $y$ respectively. |
| Game rule: | $x$ decides $\{p_1^x, p_2^x\}$ first. $y$ then decides $\{p_1^y, p_2^y\}$ after knowing $\{p_1^x, p_2^x\}$. |

**Follower's Problem:**

For any given $\widetilde{S^x}$, the follower node $y$ solves:

$$\hat{S^y}(\widetilde{S^x}) = \arg\max \ T_y(\widetilde{S^x}, \hat{S^y})$$
$$\texttt{Subject to}: C_y(\widetilde{S^x}, \hat{S^y}) \leq B_y.$$

**Leader's Problem:**

The leader node $x$ solves:

$$\hat{S^x} = \arg\min \ T_y(\hat{S^x}, \hat{S^y}(\hat{S^x}))$$
$$\texttt{Subject to}: C_x(\hat{S^x}, \hat{S^y}(\hat{S^x})) \leq B_x.$$

### *C. Two Stackelberg Equilibrium Regions*

By backward induction, we solve the above adversary Stackelberg game for nodes who have the same budget constraints, i.e., $B_x = B_y$.

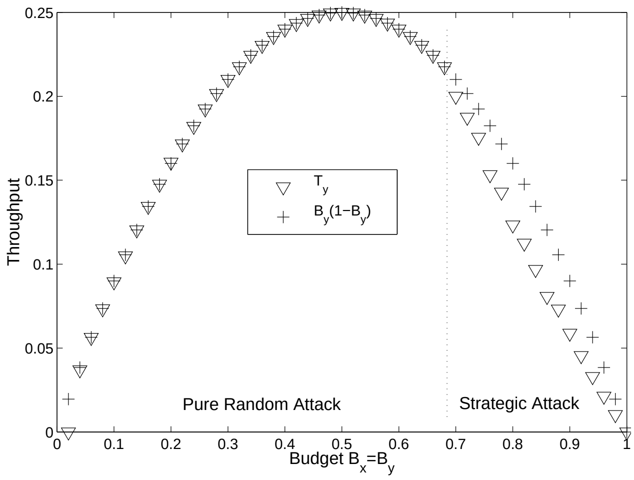



Fig. 10. Throughput in (adversary) Stackelberg equilibrium.

Figure 10 plots the throughput of the follower (non-attacking) node $y$ when $x$ chooses the optimal attacking strategy of the Stackelberg game. It also plots the curve $B_y(1 - B_x)$, which gives the throughput of node $y$ when the attacker uses a $p$-pure random attack with $p = B_x$. Figure 11 shows the costs incurred by both players. We identify two regions in the Stackelberg equilibrium solutions:

1) When the budget is less than $2/3$, both players use up their budgets. Player $y$ achieves identical throughput when attacked by the adversarial leader player and by a $p$-pure random attacker.
2) When the budget is larger than $2/3$, player $y$ achieves slightly but observably lower throughput when attacked by the adversarial leader player than attacked by the $p$-pure attacker.

Intuitively, the attacking node will always use up its budget to attack. But surprisingly, a strategic, two-state attack cannot do better than pure random attack if the adversary node does not have a budget larger than $2/3$. When the budget is larger than $2/3$, the two-state attack is only slightly more effective.

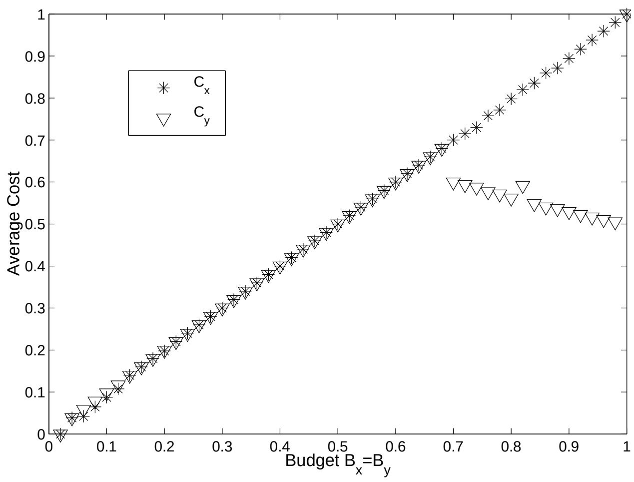
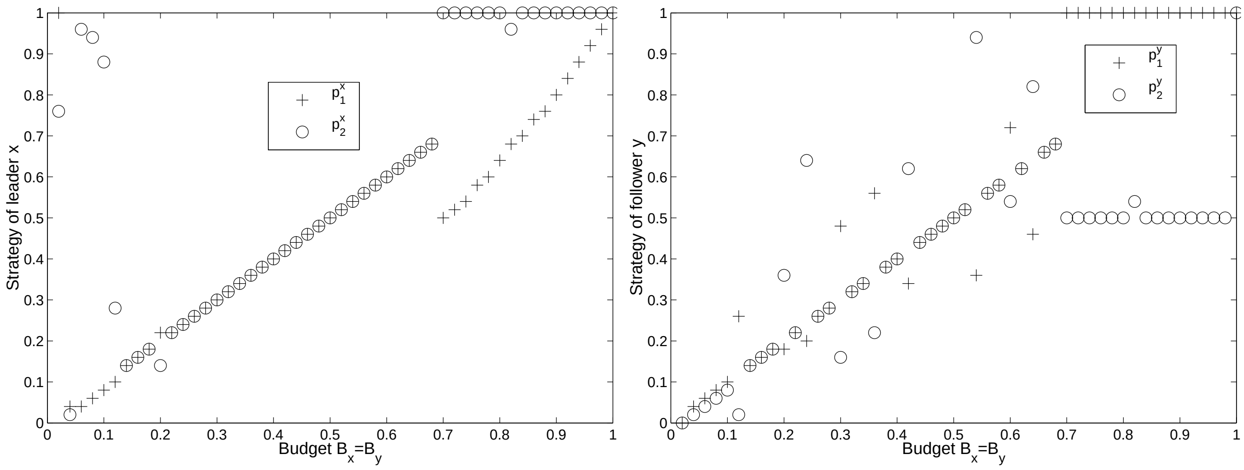



Fig. 11. Cost in (adversary) Stackelberg equilibrium.




(a) Leader's Strategy

(b) Follower's Strategy

Fig. 12. Strategies in Stackelberg equilibrium.

### *D. Random Attack Vs. Strategic Attack*

We show the strategy solutions of both players in Figure 12. We find that the strategies played in the two budget regions are quite different.

Not surprisingly, when the budget is less than $2/3$, the attacking node uses the pure random strategy $p_1^x = p_2^x = B_x$. Theorem 7 explains why the throughput $T_y$ is so close to curve $B_y(1 - B_x)$ in the lower budget region. It turns out that player $y$ has multiple strategies to maximize its throughput, but all of these strategies use up the budget $B_y$. Therefore, although the strategies played by node $y$ seem to be irregular, node $y$ always gains a throughput that is close to $B_y(1 - B_x)$. Notice that when the budget is very small (e.g. $N \leq 0.2$), the probability to have a collision is extremely small. Therefore, there are multiple optimal strategies used to maximize throughput. Of course, $p_1^x = p_2^x = B_x$ should be one of the optimal strategies that maximizes throughput.

After comparing the strategies played by both nodes in the larger budget region with those used by two non-cooperative, non-attacking nodes in Figure 8, we notice that they are strikingly similar. This means that an adversary node $x$ chooses a strategy very similar to what is chosen by a node who wishes to selfishly maximize its own throughput. Of course, node $y$ would therefore use the same response strategy.

In conclusion, if bandwidth requirements/capabilities are low, an attacker cannot do much better than attacking at random points in time. If the bandwidth requirements and capabilities are high, then an attacker behaves similarly to a node seeking to maximize its own throughput.

## VIII. CONCLUSION

In this paper, we generalize the slotted-Aloha protocol to a two-state protocol and construct a Markov Model for it. We find that if all nodes cooperate in an effort to maximize the aggregate throughput, an aggregate throughput of at least one half

($\rho > 1/2$) can be achieved regardless of the number of nodes competing for bandwidth. If all nodes are selfish and attempt to maximize their own individual throughput, a situation similar to the traditional Prisoner's Dilemma arises. Specifically, for a two-node system with budget constraints, the solution has the following features: (1) When each node's transmitting budget is extremely limited, an aggressive strategy maximizes each individual node's throughput as well as the aggregate system throughput. (2) When each node's transmitting budget is in a medium range, an aggressive strategy produces a local maximum throughput, but a cooperative strategy would have produced higher throughput. (3) When each node's budget is in the upper range, a node's optimal strategy depends heavily on what its competitors choose to do. Finally, we showed that adversary nodes with limited budgets can do little better than a random attack, and nodes with large budgets should behave like their selfish counterparts.

The generalized slotted-Aloha provides a framework to analyze different behavior of autonomous nodes in system. Some analytical observations from different behavioral modelings provide guidelines for building robust and efficient media access protocols, from which systems can obtain higher aggregate throughput, as well as the ability to cope with selfish and malicious users.

## APPENDIX I
## PROOF OF OPTIMALITY

In this appendix, we prove that for a homogeneous system with $N$ nodes, the strategy $(p_1 = 1, p_2 \to 0$ is optimal. As we showed that the achievable throughput is $\frac{N}{2N-1}$, we need to prove that it is also the upper bound for the system for any strategy $(p_1, p_2)$.

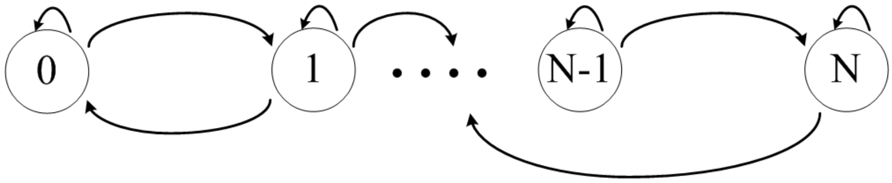



Fig. 13. Original system with $N+1$ states.

We model our system as a Markov chain $S$ with $N+1$ states in figure 13. Each state $i$ of the Markov chain indicates that there are $i$ nodes in free-state and $N-i$ nodes in backlogged-state. We notice that the forward transition happens only from state $i$ to $i+1$ because at most one backlogged node can change to a free-state node during a time-slot.

We will complete the proof in three steps. In the first step, we want to show in Lemma 5 that if the system is in states other than 1 and $N-1$, the instantaneous throughput should be no more than $1/2$. The intuition is that when multiple nodes (more than one node) are using the same transmitting probability in a time-slot, the throughput will be no more than $1/2$.

**Lemma** *2:* Let $X$ and $Y$ be two independent indicator R.V.s with $x = P(X = 1), y = P(Y = 1)$. If $x, y < 1/2$, or $x, y > 1/2$, then $P(X \oplus Y = 1) < 1/2$. If $x = 1/2$ or $y = 1/2$, then $P(X \oplus Y = 1) = 1/2$.

*Proof:* By contradiction, assume $P(X \oplus Y = 1) \geq 1/2$. Since $X$ and $Y$ are independent, we have $P(X \oplus Y = 1) = x + y - 2xy \geq 1/2$. For the case where $x, y < 1/2$, noting $1 - 2y > 0$, we get that $x(1-2y) \geq 1/2 - y$ reduces to $x \geq 1/2$, yielding the contradiction. A contradiction follows similary when $x, y > 1/2$ by noting that $1 - 2y < 0$. The final equality follows from simple algebra by setting $y = 1/2$. ■

**Lemma** *3:* Consider a finite series $\{X_1, \cdots, X_N\}$ of independent indicator R.V.s where $P(X_i = 1) < 1/2$ for all $i$, then $P(\sum_{i=1}^{N} X_i = 1) < 1/2$ i.e., there is less than probability of $1/2$ that exactly one r.v. equals 1. Furthermore, "$<$" can be replaced by "$\leq$" everywhere.

*Proof:* We The proof is by induction. By Lemma 2, the result holds for $N = 2$. Assuming the result holds true for $N-1$, let $Z$ be an indicator r.v. that equals 0 unless $\sum_{i=1}^{N-1} X_i = 1$. Note that for $\sum_{i=1}^{N} X_i = 1$ it is necessary (but not sufficient) that either $Z = 1$ and $X_N = 0$ or $Z = 0$ and $X_N = 1$, hence $P(\sum_{i=1}^{N} X_i = 1) < P(Z = 1 \oplus X_N = 1)$. By the inductive hypothesis, $P(Z = 1) < 1/2$, and $Z$ is independent of $X_N$. Applying Lemma 2 to r.v.s $Z$ and $X_N$ yields the result. ■

**Lemma** *4:* Consider a finite series $\{X_1, \cdots, X_N\}$ of independent indicator R.V.s where $P(X_i = 1) > 1/2$ for all $i$, then $P(\sum_{i=1}^{N} X_i = 1) < 1/2$ i.e., there is less than probability of $1/2$ that exactly one r.v. equals 1. Furthermore, if the $X_i$ can equal $1/2$, then $P(\sum_{i=1}^{N} X_i = 1) \leq 1/2$.

*Proof:* The case for $N = 2$ follows directly from Lemma 2. For $N = 3, P(X_1 + X_2 + X_3 = 2) = P(X_1 + X_2 = 1)P(X_3 = 0) + P(X_1 + X_2 = 0)P(X_3 = 1)$. Applying Lemma 2 to the first term yields the solution to be smaller than $(1/2)(1/2) + 1/4P(X_3 = 1) < 1/2$. We prove $N > 3$ inductively by letting $Y$ and $Z$ be indicator r.v.s that respectively equal 1 when $\sum_{i=1}^{N-2} X_i = 1$ and when $X_{N-1} + X_N = 1$. Note that $\sum_{i=1}^{N} X_i = 1$ is synonymous with $Y \oplus Z = 1$). By the inductive hypothesis, $P(Y = 1) < 1/2$ and $P(Z = 1) < 1/2$, and since $Y$ and $Z$ are independent, the proof follows from Lemma 2 applied to $Y$ and $Z$. ■

**Corollary 1:** If a set of multiple independent, indicator random variables must choose identical probabilities, then the probability that exactly one of the r.v.s evaluates to 1 is no greater than $1/2$.

*Proof:* Follows immediately from Lemmas 3 and 4. ■

We can now focus on some properties of the Markov Chain $S$. We say the throughput of a set of states $\tau$ is the conditional probability that a transmission is successful given that the system is in a state of $\tau$, and we write $T(\tau)$ to indicate this throughput. When $\tau = \{i\}$, a single state, we simply write $T(i)$. $T(S)$ represents the throughput of the entire system (i.e., all the states).

**Lemma** *5:* $T(i) \leq 1/2$ for $i = 0, N$ and $1 < i < N-1$, i.e., all states except 1 and $N-1$.

*Proof:* States 0 and $N$ follow immediately from Corollary 1, where each $X_i$ indicates whether node $i$ transmitted, and $P(X_i = 1) = p_2$ and $p_1$ respectively for states 0 and $N$.

For the remaining states, the nodes are split into two groups, free and backlogged. Each group must use the same transmission probability and each group has more than 1 node within it. For each group, we apply Corollary 1 to show that, regardless of the values chosen for $p_1$ and $p_2$, the likelihood of a single success in each group is no larger than $1/2$. We then apply Lemma 2 to show that the likelihood of a single transmission out of the two groups is no larger than $1/2$. ■

In the second step of the proof, we want to show that if the system throughput $T(S)$ is greater than $1/2$, one of the following conditions must be satisfied:

- $p_1 > 1/2$ and $p_2 < 1/2$, which is the necessary condition for $T(1) > 1/2$.
- $p_1 < 1/2$ and $p_2 > 1/2$, which is the necessary condition for $T(N-1) > 1/2$.

This result comes from Lemma 5, which implies that the system throughput can be greater than $1/2$ only if $T(1) > 1/2$ and/or $T(N-1) > 1/2$.

**Lemma** *6:* $T(N-1) > 1/2$ requires that $p_2 > 1/2$ and $p_1 < 1/2$.

*Proof:* There is more than one node in the free state that must all have the same transmission probability, so by Corollary 1, the likelihood of only one transmission from this group is no greater than $1/2$. If $p_2 \leq 1/2$, it then follows from Lemma 2 (with $X$ representing a single transmission form the group of free nodes and $Y$ from the backlogged node) that a single transmission succeeds over all nodes with probability no greater than $1/2$. Hence, $p_2 > 1/2$.

If $p_1 \geq 1/2$, then all nodes transmit with probability no less than $1/2$, and it follows from Lemma 4 that a single transmission succeeding occurs with probability no greater than $1/2$. So to achieve the Lemma, we must also have $p_1 < 1/2$. ■

The proof is the same (swapping discussion of free nodes with backlogged nodes) for the following:

**Lemma** *7:* $T(1) > 1/2$ requires that $p_1 > 1/2$ and $p_2 < 1/2$.

Putting together Lemmas 6 and 7, we get

**Corollary 2:** The system $S$ cannot simultaneously have $T(1) > 1/2$ and $T(N-1) > 1/2$.

In the third step of the proof, we want to prove that under each of the conditions $T(1) > 1/2$ or $T(N-1) > 1/2$, the system throughput cannot be greater than or equal to $\frac{N}{2N-1}$. Therefore, the system throughput is upper bounded by $\frac{N}{2N-1}$, and $p_1 = 1, p_2 \rightarrow 0$ is optimal.

For each of the two conditions, we reduce the original Markov chain $S$ into a simplified system. Using sample path argument, we show that the throughput of the reduced system is no less than the original system. After that, we prove that the throughput of each reduced system is upper bounded by $\frac{N}{2N-1}$.

We first consider the condition when $T(1) > 1/2$. We reduce the original Markov into a reduced system $S'$, which has only two states (state 0 and state 1), in figure 14.

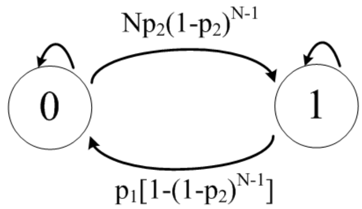



Fig. 14. Reduced system $S'$ with state 0 and 1.

We build the reduced system $S'$ by turning all transitions from state 1 to a state $i > 1$ to turn back to state 1. Our claim is that under the condition $T(1) > 1/2$, if $T(S) > N/(2N-1)$, then $T(S') > T(S)$ as well. This can be shown via sample-path analysis. In short, each sample path $\phi$ in $S$ is converted into a path $\phi'$ in $S'$ by removing the intervals that starts where the path transitions from state 1 to state 2, and ends when the system finally returns to state 1. Since the removed portion does not include entry into state 1, the expected throughput contribution of removed portions is less than $1/2$. For any $\phi'$, the memoryless property of the system ensures that the probability of one of the $\phi'$ that maps to $\phi$ occuring in $S$ equals the probability that $\phi'$ occurs in $S'$. The following Lemma 8 can then be applied with $C = N/(2N-1)$ to show that if $T(S) > N/(2N-1)$, then the expected throughput of the truncated parts must exceed $N/(2N-1)$ as well, and hence $T(S') > N/(2N-1)$.

**Lemma** *8:* Consider a series of variables $x_0, \cdots, x_N$ and a series of variables $y_0, \cdots, y_N$ where $x_0/y_0 < C$ yet $\sum_{i=0}^{N} x_i / \sum_{i=0}^{N} y_i \geq C$. Then $\sum_{i=1}^{N} x_i / \sum_{i=1}^{N} y_i \geq C$ as well.

*Proof:* Let $a = \sum_{i=1}^{N} x_i$ and $b = \sum_{i=1}^{N} y_i$. Then the above lemma states that $(x_0 + a)/(y_0 + b) \geq C$ while $x_0/y_0 < C$. We thus have $x_0 + a \geq C(y_0 + b)$. Since $x_0 < Cy_0$ , it must be the case that $a \geq Cb$ for the above inequality to hold. ■

**Lemma** *9:* The throughput of the reduced system $T(S')$ is less than $\frac{N}{2N-1}$.

*Proof:* Let us focus on the reduced system $S'$ with $N$ nodes. The transition probability from state 0 to state 1 is

$$p_{01} = Np_2(1-p_2)^{N-1}.$$

The transition probability from state 1 to state 0 is

$$p_{10} = p_1[1-(1-p_2)^{N-1}].$$

We can solve the steady-state distribution for this two-state Markov chain.

$$\pi_0 = \frac{p_{10}}{p_{01}+p_{10}}, \pi_1 = \frac{p_{01}}{p_{01}+p_{10}}.$$

The throughput is

$$T(S') = \pi_0 p_{01} + \pi_1[p_1(1-p_2)^{N-1} + (1-p_1)(N-1)p_2(1-p_2)^{N-2}].$$

$$= \frac{Np_2(1-p_2)^{N-1}}{p_1 + (Np_2 - p_1)(1-p_2)^{N-1}}[p_1 + (1-p_1)(N-1)p_2(1-p_2)^{N-2}].$$

$$= Np_2(1-p_2)^{N-1}\frac{p_1 + (1-p_1)(N-1)p_2(1-p_2)^{N-2}}{p_1 + (Np_2 - p_1)(1-p_2)^{N-1}}.$$

$$= Np_2(1-p_2)^{N-1}\frac{1-(N-1)p_2(1-p_2)^{N-2} + \frac{1}{p_1}(N-1)p_2(1-p_2)^{N-2}}{1-(1-p_2)^{N-1} + \frac{1}{p_1}Np_2(1-p_2)^{N-1}}.$$

Let $a = 1-(N-1)p_2(1-p_2)^{N-2}$,$b = (N-1)p_2(1-p_2)^{N-2}$,$c = 1-(1-p_2)^{N-1}$, and $d = Np_2(1-p_2)^{N-1}$, we have

$$T(S') = [Np_2(1-p_2)^{N-1}]\frac{a + \frac{1}{p_1}b}{c + \frac{1}{p_1}d}.$$

All $a, b, c$ and $d$ are non-negative. We test the quantity $\frac{a}{c} - \frac{b}{d}$.

$$\frac{a}{c} - \frac{b}{d} = \frac{ad - bc}{cd} = \frac{1}{cd}N(1-p_2)[1-(N-1)p_2(1-p_2)^{N-2}] - (N-1)[1-(1-p_2)^{N-1}]$$

$$= \frac{1}{cd}(1-Np_2)[1 + (N-1)(1-p_2)^{N-1}]$$

The condition for $\frac{a}{c} \leq \frac{b}{d}$ is $p_2 \geq 1/N$. Therefore, the maximum throughput is achieved at $p_1 = 0$, where $T(S') = (N-1)p_2(1-p_2)^{N-2} \leq (1-\frac{1}{N-1})^{N-2} \leq 1/2 \; \forall \; N > 2$.

The condition for $\frac{a}{c} > \frac{b}{d}$ is $p_2 < 1/N$. In that case, the maximum throughput is achieved when $p_1 = 1$.

$$T(S') = \frac{Np_2(1-p_2)^{N-1}}{1 + (Np_2 - 1)(1-p_2)^{N-1}} = \frac{N}{N + \frac{1}{p_2(1-p_2)^{N-1}} - \frac{1}{p_2}}.$$

We want to prove that the above is no more than $\frac{N}{2N-1}$. It is equivalent to show the following:

$$\frac{1}{p_2(1-p_2)^{N-1}} - \frac{1}{p_2} \geq N - 1$$

After arranging the terms, we want to show

$$f(p_2) = [(N-1)p_2 + 1](1-p_2)^{N-1} \leq 1$$

$$f'(p_2) = (N-1)(1-p_2)^{N-1} - [(N-1)p_2 + 1](N-1)(1-p_2)^{N-2}$$

$$= -Np_2(N-1)(1-p_2)N - 2 \leq 0$$

Therefore, the maximum of $f(p_2)$ is achieved at $p_2 = 0$ where $f'(p_2) = 0$. $f(p_2)^* = f(0) = 1 \leq 1$.

■

Then, we consider the condition when $T(N-1) > 1/2$. We reduce the original Markov into a reduced system $S''$, which also has only two states (state $N-1$ and state $N$), in figure 15.

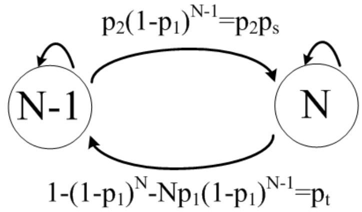



Fig. 15. Reduced system $S''$ with state $N-1$ and $N$.

We build the reduced system $S''$ by turning all transitions from state $N$ to a state $i < N$ to turn back to state $N-1$, and all transitions from state $N-1$ to a state $i < N-1$ to turn back to state $N-1$. Our claim is that under the condition $T(N-1) > 1/2$, if $T(S) > N/(2N-1)$, then $T(S'') > T(S)$ as well. The same sample-path analysis applies.

**Lemma** *10:* The throughput of the reduced system $T(S'')$ is less than $\frac{N}{2N-1}$.

*Proof:* We define the transition probability from state $N-1$ to $N$ to be

$$p_{N-1,N} = p_2(1-p_1)^{N-1} = p_2 p_s.$$

This is the probability that the backlogged node transmits and none of the free-state node transmits.

When the system is in state $N$, it stays with it with probability $(1-p_1)^N + Np_1(1-p_1)^{N-1}$. This is the probability that none of the nodes transmits or only one of the nodes transmits. Therefore, for the reduced system, we have the transition probability from state $N$ to $N-1$ to be

$$p_{N,N-1} = 1-(1-p_1)^N - Np_1(1-p_1)^{N-1} = p_t.$$

The steady-state distribution for the reduced system is

$$\pi_{N-1} = \frac{p_t}{p_t + p_2 p_s}, \pi_N = \frac{p_2 p_s}{p_t + p_2 p_s}.$$

Therefore, the throughput is

$$T(S'') = \pi_{N-1}[p_2 p_s + (1-p_2)(N-1)p_1(1-p_1)^{N-2}] + \pi_N[Np_1(1-p_1)^{N-1}].$$

$$= \frac{p_t(p_2 p_s + (1-p_2)(N-1)p_1(1-p_1)^{N-2})}{p_t + p_2 p_s} + \frac{p_2 p_s N p_1(1-p_1)^{N-1}}{p_t + p_2 p_s}.$$

$$= \frac{p_t(N-1)p_1(1-p_1)^{N-2} + [p_t p_s - p_t(N-1)p_1(1-p_1)^{N-2} + p_s N p_1(1-p_1)^{N-1}]p_2}{p_t + p_s p_2}.$$

Let $a = p_t(N-1)p_1(1-p_1)^{N-2}$, $b = p_t p_s - p_t(N-1)p_1(1-p_1)^{N-2} + p_s N p_1(1-p_1)^{N-1}$, $c = p_t$ and $d = p_s$, the throughput becomes $T(S'') = (a + bp_2)/(c + dp_2)$.

Let us check the value of $a/c - b/d$.

$$\frac{a}{c} - \frac{b}{d} = (1 + p_t/p_s)(N-1)p_1(1-p_1)^{N-2} - [p_t + Np_1(1-p_1)^{N-1}]$$

$$\frac{a}{c} - \frac{b}{d} = (p_t + p_s)\frac{(N-1)p_1}{1-p_1} - [1-(1-p_1)^N]$$

$$\frac{a}{c} - \frac{b}{d} = [\frac{(N-1)p_1}{1-p_1} - 1][1-(1-p_1)^N] - \frac{(N-1)p_1}{1-p_1}(Np_1 - 1)(1-p_1)^{N-1}$$

$$\frac{a}{c} - \frac{b}{d} = \frac{Np_1 - 1}{1-p_1}[1-(1-p_1)^N - (N-1)p_1(1-p_1)^{N-1}] = \frac{Np_1 - 1}{1-p_1}[p_t + p_1(1-p_1)^{N-1}]$$

We conclude that $a/c - b/d \geq 0$ if $Np_1 - 1 > 0$. In that case, the maximum throughput is achieved at $p_2 = 0$. Therefore,

$$T(S'')^* = (N-1)p_1(1-p_1)^{N-2} \leq (1 - \frac{1}{N-1})^{N-2} \leq \frac{1}{2}\ \forall\ N \geq 3.$$

The remaining case is that $a/c - b/d < 0$ when $p_1 < 1/N$. In this case, the maximum throughput is achieved at $p_2 = 1$. Then the throughput becomes:

$$T(S'')^* = \frac{p_t p_s + p_s N p_1(1-p_1)^{N-1}}{p_t + p_s}.$$

We want to prove the above is less than or equal to $\frac{1}{2}$ for all $p_1 \in [0, \frac{1}{N}]$. First, we check the two boundary cases where $p_1 = 0$ and $p_1 = \frac{1}{N}$. When $p_1 = 0$, $T(S'')^* = 0$. When $p_1 = \frac{1}{N}$, $T(S'')^* = (1 - \frac{1}{N})^{N-1} \leq \frac{1}{2}$. Finally, we want to make sure that the local maximum is not greater than $\frac{1}{2}$.

$$T(S'')^* = \frac{p_s}{p_t + p_s}[p_t + Np_1(1-p_1)^{N-1}] = \frac{p_s[1-(1-p_1)^N]}{p_t + p_s} = \frac{(1-p_1)^N[1-(1-p_1)^N]}{(1-p1)(p_t+p_s)}.$$

The numerator is less than or equal to $\frac{1}{4}$, therefore, all we want to prove is that the denominator is greater than or equal to $\frac{1}{2}$. We want to prove $(1-p1)(p_t+p_s) \geq \frac{1}{2}$.

$$\begin{aligned}(1-p1)(p_t+p_s) &= (1-p_1)[1-(1-p_1)^N] + (1-p_1)^N(1-Np_1) \\ &= 1-(1-p_1)^N - p_1 + p_1(1-p_1)^N + (1-p_1)^N - Np_1(1-p_1)^N \\ &= 1-p_1-(N-1)(1-p_1)[p_1(1-p_1)^{N-1}] \geq 1-p_1-(N-1)(1-p_1)[\frac{1}{N}(1-\frac{1}{N})^{N-1}] \\ &= (1-p_1)[1-\frac{N}{N-1}(1-\frac{1}{N})^{N-1}] = (1-p_1)[1-(1-\frac{1}{N})^N] > (1-\frac{1}{N})[1-(1-\frac{1}{N})^N]\end{aligned}$$

When $N = 4$, the above equals $(1-\frac{1}{4})(1-(\frac{3}{4})^4) > \frac{1}{2}$. For $N > 4$,

$$(1-\frac{1}{N})[1-(1-\frac{1}{N})^N] > (1-\frac{1}{N})(1-e^{-1}) > (1-\frac{1}{N}) * 0.63 > \frac{1}{2}.$$

The last case to show is that when $N = 3$, $T_3^*(S'') = \frac{(1-p_1)^2[1-(1-p_1)^3]}{1-(1-p_1)^3+(1-3p_1)(1-p_1)^2} \leq \frac{1}{2}$.

Let $x = 1-p_1$ and $f(x) = \frac{1}{T_3^*(S'')}$. We have

$$f(x) = \frac{1}{T_3^*(S'')} = \frac{1}{(1-p_1)^2} + \frac{1-3p_1}{1-(1-p_1)^3} = \frac{1}{x^2} + \frac{3x-2}{1-x^3}$$

$$f'(x) = \frac{-2}{x^3} + \frac{3}{1-x^3} + \frac{(3x-2)3x^2}{(1-x^3)^2}$$

Set the above first derivative to zero, we get the condition for $x^* = 1-p_1^*$ when the $T_3^*(S'')$ is maximized.

$$6(x^*)^5 = 4(x^*)^6 + 7(x^*)^3 - 2.$$

We want to prove $f(x) \geq 2$, which is equivalent to show the following:

$$\frac{1}{x^2} + \frac{3x-2}{1-x^3} \geq 2$$

$$6x^5 \geq -6x^3 + 12x^2 - 3$$

Substitute the first derivative condition, we need to show

$$4(x^*)^6 + 7(x^*)^3 - 2 \geq -6(x^*)^3 + 12(x^*)^2 - 3$$

$$10(x^*)^3 + 5(x^*)^2 + 1 > 0$$

■

By using the above two lemmas, we showed that under no circumstances that the throughput $T(S)$ can be higher than $\frac{N}{2N-1}$. Thus, $p_1 = 1, p_2 \to 0$ is optimal since it converges to the throughput upper bound $\frac{N}{2N-1}$.

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