On the Optimality of Greedy Garbage Collection for SSDs

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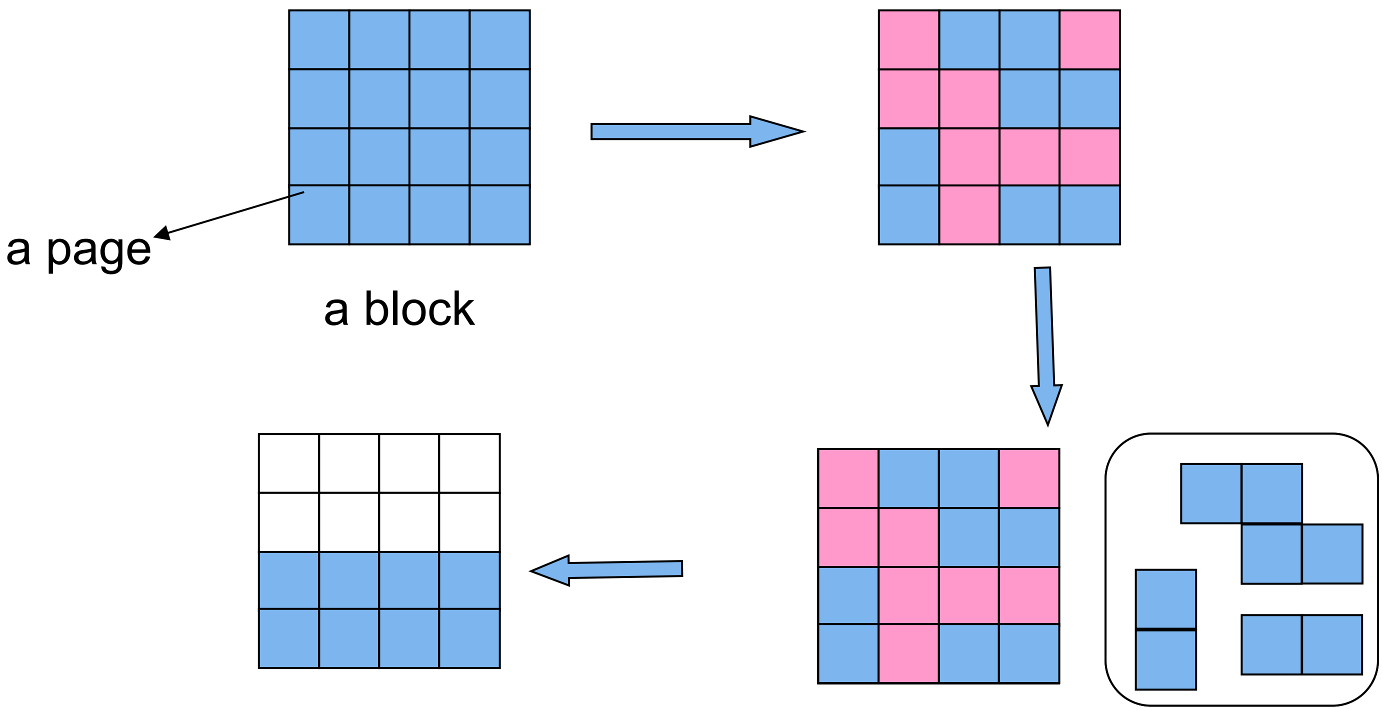
SSD Introduction

- Block
- Page
- Page has three states: valid, invalid, and empty



- write 16 pages for 8 write requests
  → write-amplification = 2

System Model

- N blocks, B pages in each block
- write-amplification: $\frac{\text{overhead} + \text{data}}{\text{data}}$,or $lim_{M\to\infty} \frac{MB}{\sum_{j=1}^{M} i_j}$

  M : number of cleans

  i_j : number of invalid pages on the j-th clean

smaller write-amplification→better performance

Goal: Minimize write-amplification

Related works

- Greedy algorithm Hu 2006, Bux 2010, Desnoyers 2012
  - *The performance of greedy algorithm is analyzed*
- Dual-greedy Lin 2012
  - *A heuristic algorithm optimized for traced data*
- D-choice Van 2013, Li 2013
  - *D-choice algorithm is proposed and analyzed*
- Optimality of greedy Hass 2010
  - *A sketch proof of optimality of greedy on random write workload.*

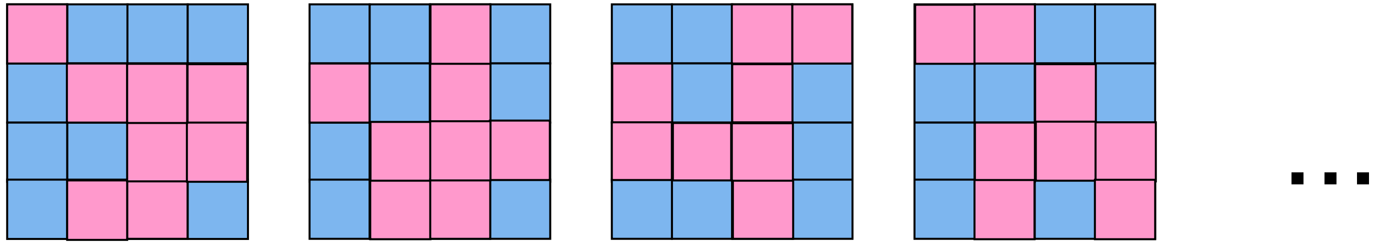
Contribution

First formal proof of the optimality of greedy algorithm on memoryless workloads.

Memoryless workloads

All valid pages have equal probability of becoming the next invalid page.

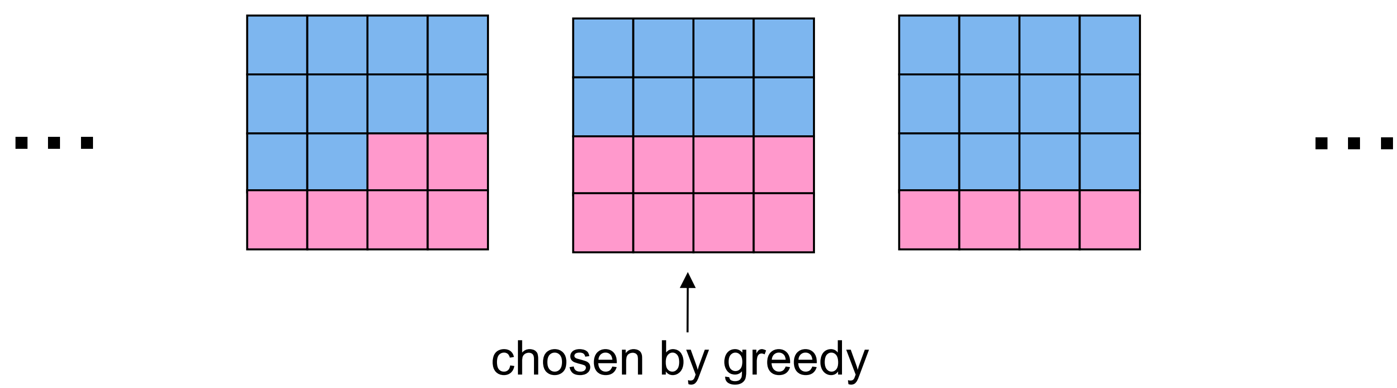
– Page lifetimes: independent, exponential with identical rate μ



Greedy GC Algorithm

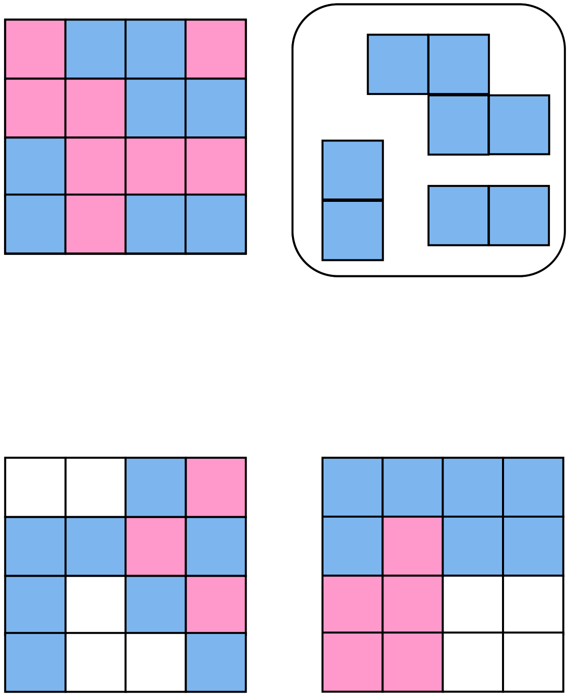
The Greedy GC algorithm:

- waits until the SSD is filled.
- cleans a block with maximal number of invalids.
- ties are broken arbitrarily.



Clean-and-move system

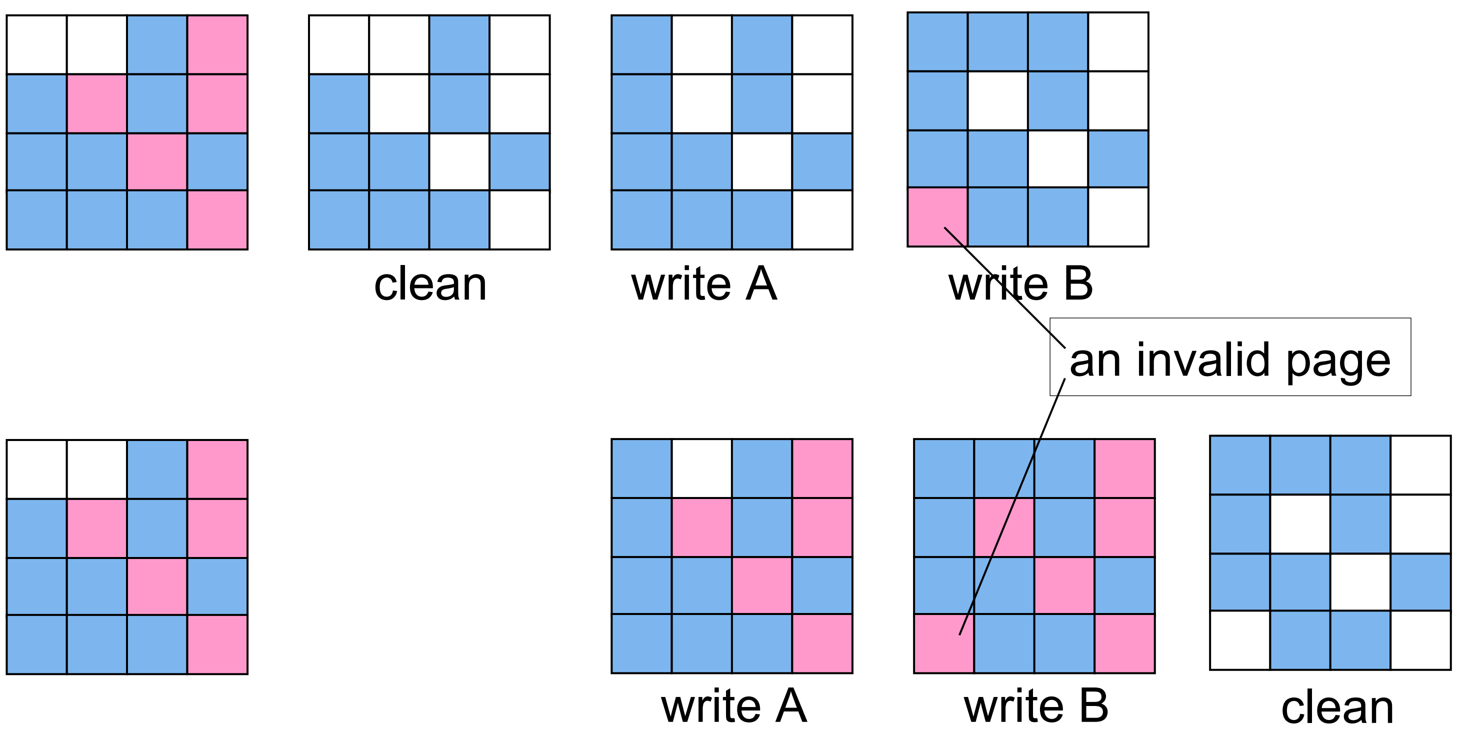
- allows the valid pages to write on other blocks.



Optimality of Greedy

- Lemma 1. A block should only be cleaned when it is full (of valid and invalid pages).
- Proof:

  Cleaning can always be delayed while the block containing empties.

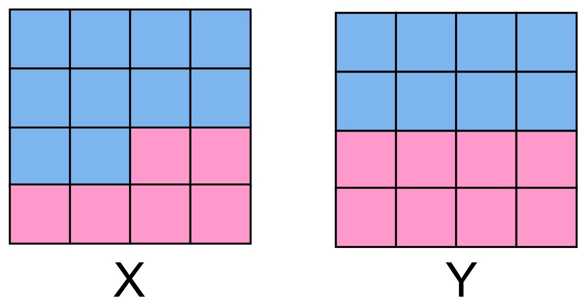


Optimality of Greedy

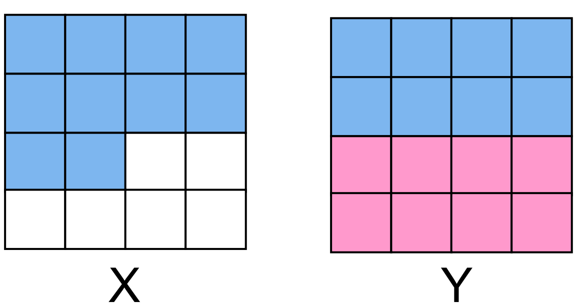
Theorem 1. In clean-and-move systems with memoryless workloads, Greedy is optimal.

Proof: X: the block choose by another algorithm

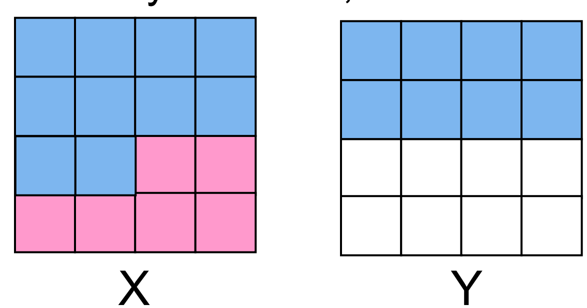
Y: the block choose by greedy



Algorithm $\mathcal{A}$ cleans X



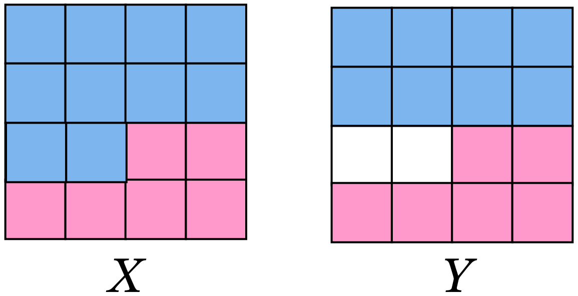
Greedy cleans Y, and moves



Optimal of Greedy

Theorem 2. For memoryless workloads, there is no advantage to moving active pages between blocks.

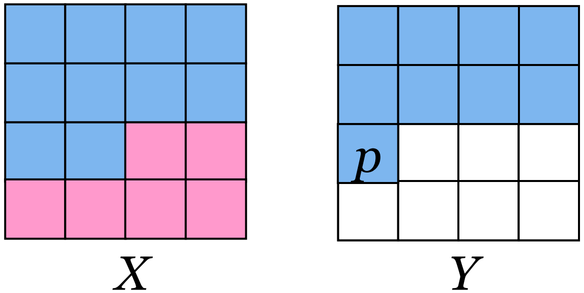
Proof: Suppose we are moving n pages from X to Y upon an write request p



Y must has exact n empty pages, otherwise we can delay move.

If p is placed on Y: *move* $\rightarrow$ clean $Y \rightarrow$ write p

The move operation can be delayed, so the sequence become clean $Y \rightarrow$ write $p \rightarrow$ move(upon next request)



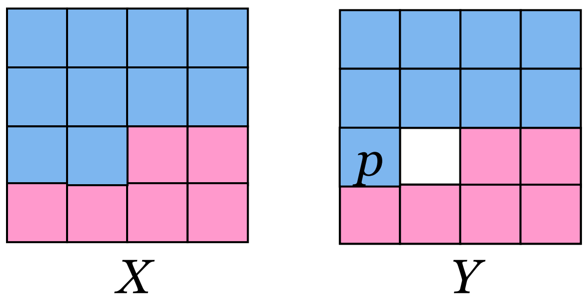
Optimal of Greedy

X must has exact no empty pages, otherwise we can delay move

If p is placed on X: *move* n *pages from* X *to* Y $\rightarrow$ clean X $\rightarrow$ write p in X

delay the move by a stochastic equivalence:

write p in Y $\rightarrow$ *move* n-1 *pages from* X *to* Y (upon later arrival)



Discussion & Future work

- D-choice variants: choosing a block from a randomly selected set of size D

  Greedy still be the optimal on D-choice variants.
- more general workloads
  - not optimal for Rosenblum workloads and long-tailed workloads. Lin 2012, Van 2013
  - conjecture: still optimal for short-tailed workloads.
- Heuristics for general workload