Big Winners and Small Losers of Zero-rating

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An objective of network neutrality is to design regulations for the Internet and ensure that it remains a public, open platform where innovations can thrive. While there is broad agreement that preserving the content quality of service falls under the purview of net neutrality, the role of differential pricing, especially the practice of *zero-rating*, remains controversial. Zero-rating refers to the practice of providing free Internet access to some users under certain conditions, which usually concurs with differentiation among users or content providers. Even though some countries (India, Canada) have banned zero-rating, others have either taken no stance or explicitly allowed it (South Africa, Kenya, U.S.).

In this article, we model zero-rating between Internet service providers and content providers (CPs) to better understand the conditions under which offering zero-rating is preferred, and who gains in utility. We develop a formulation in which providers' incomes vary, from low-income startups to high-income incumbents, where their decisions to zero-rate are a variation of the traditional prisoner's dilemma game. We find that if zero-rating is permitted, low-income CPs often lose utility, whereas high-income CPs often gain utility. We also study the competitiveness of the CP markets via the *Herfindahl Index*. Our findings suggest that in most cases the introduction of zero-rating *reduces* competitiveness.

CCS Concepts: • **Networks** → **Network economics**; • **Theory of computation** → *Network games*; • **Computing methodologies** → Model verification and validation; • **Social and professional topics** → **Net neutrality;**

Additional Key Words and Phrases: Differential pricing, zero rating, network neutrality

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1 INTRODUCTION

Net neutrality is the principle that **Internet service providers (ISPs)** treat all data on the Internet equally and do not discriminate or charge differently by user, content, website, platform, type of equipment, or method of communication [1, 2]. Net neutrality advocates claim that such discrimination weakens **content providers' (CP)** investment incentives, since ISPs can expropriate some of the investment benefits [3, 4]. However, net neutrality opponents argue that *differential pricing*,

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which allows ISPs to charge *end users* different prices for data originating from various websites (CPs), triggers investments in broadband capacity and content innovation [5]. Although by some definitions, net neutrality does not include issues involving pricing, it is worthwhile to study how differential pricing impacts the market and how data is ultimately treated, where net-neutrality centers (i.e., user content, etc.).

There is no legislation directly addressing net neutrality in the U.S. However, net neutrality can sometimes be enforced based on other laws, such as those preventing anti-competitive practices. In October 2009, the **Federal Communications Commission (FCC)** proposed draft rules for "preserving a free and open Internet" to justify this enforcement based on compliance with "commercially reasonable" practices [6]. The FCC has therefore elected to examine on a case-by-case basis that "prohibits unreasonable interference with end users' ability to select content and CPs' ability to reach end-users" [7]. In June 2018, FCC released Open Internet Order to restore Internet freedom completely [8]. India, however, passed the world's *strongest* net neutrality regulations in 2016 [9], prohibiting differential pricing and thus by implication zero-rating, followed by a similar ruling in Canada. Their rulings follow a definition of net neutrality that states "the Internet should be maintained as an open platform, on which network providers treat all content, applications, and services equally, without discrimination." This ruling includes ensuring that network providers do not supply any competitive advantage to specific apps/services, either through pricing or Quality of Service [10].

A commonly used practice of differential pricing is *zero-rating*: a service where ISPs do not charge customers for bandwidth consumed by specific applications and services, while customers pay the bandwidth price for other used services. Today, zero-rating is used in practice by particular cellular network providers [11]. In the U.S. since June 2014, the mobile service provider T-Mobile has offered zero-rated access for participating music streaming services to its mobile Internet customers [12, 13]. In November 2015, they expanded zero-rated access to video streaming services [14]. In 2016, Verizon joined T-Mobile by creating its own sponsored data program, FreeBee Data, which is an example of a zero-rated service [15].

Proponents of zero-rating argue that offering some services for free increases customer satisfaction and online services usage, as they can access more data at a given cost [16]. Critics argue that zero-rating leads to settings where sponsored data allows more financially prosperous CPs to pay for placement, which adversely affects smaller CPs who cannot afford the same luxury [12, 15]. From this perspective, zero-rating may create artificial scarcity and jeopardize the achievement of the net neutrality rationale [17].

Arguments to date, such as the above, are qualitative, and there is no formal model that would allow concerned parties to quantitatively ascertain how zero-rating impacts the marketplace. Thus, there is no way to quantify the extent of the win or loss in a heterogeneous market of ISPs, CPs, and users based on enabling or disabling a zero-rated service.

In this article, we extend the work in Reference [18] and formally model Internet settings where zero-rating is offered by **ISPs** to the **CPs** who deliver their content to **users**. We build our models to analyze how zero-rating impacts the market when ISPs, CPs, and consumers choose options that maximize their individual rewards. We perform (1) a **macroscopic** analysis, i.e. zero-rating impacts on the competitiveness of the market as a whole, (2) a **microscopic** analysis, i.e., zero-rating impacts on the behavior and decisions of individual ISPs and CPs.

Our model differentiates CPs in terms of their *value*, i.e., how much revenue a CP makes per bandwidth unit used by their customers. Our model is most valid when CPs that we study provide a similar type of date, e.g., all streaming, or all social media. Therefore, a CP that is more popular among users is more prosperous and has a higher value than its counterparts. Incumbents typically have higher values, but startups have lower values, making less money per unit of bandwidth due to their smaller size and smaller market popularity. While our model and theoretical results are general for *any* kind of differential pricing, in this article, we tailor our analysis explicitly to the zero-rating context, as it is the only prevalent real-world implementation of late. The new knowledge can help providers make informed zero-rating decisions and guide regulators to design better policies to address net neutrality issues from an interconnection context. Our contributions and conclusions are as follows.

- We consider both ISPs' and CPs' zero-rating decisions as a bargaining problem, analyze their strategic behavior, and introduce the concept of *zero-rating equilibria.*
- We identify the *zero-rating pressure* phenomenon where CPs only zero-rate when their competitors do, and find how it impacts CPs' decisions and their utilities.
- We analyze the impact of zero-rating on the market of CPs globally by evaluating the *Herfindahl index* [19, 20], which is a commonly accepted measure of market concentration and is used to determine market competitiveness, and show that zero-rating availability in a heterogeneous market of CPs increases market distortion by decreasing competition.
- We analyze the impact of zero-rating individually by evaluating CPs' utilities, and show that zero-rating mostly impacts the utility of low-value CPs negatively and the utility of high-value CPs positively.
- We numerically explore the parameter space of our model and demonstrate the impact of zero-rating on market shares and profitability of the CPs under varying market conditions.

The rest of the article is organized as follows. Section 2 builds the choice model, which takes ISPs and CPs as complementary services and characterizes their market shares (Equation (2)). Section 3 builds the utility model (Equation (4)) under various zero-rating and market structures. Section 4 theoretically analyzes the Herfindahl index and utilities under zero-rating equilibria, and Section 5 numerically measures the Herfindahl index and CPs' utilities under the equilibria in a duopolistic market of ISPs/CPs. Section 6 presents the related work, and the article is concluded in Section 7.

2 MODEL

We consider a setting with three types of players: *user, ISP,* and *CP*. We denote the set of ISPs by $\mathbb{M}$ and CPs buy $\mathbb{N}$. To receive content, a user selects one of $|\mathbb{M}|$ ISPs as their bandwidth provider and can choose from one or more of the $|\mathbb{N}|$ CPs as their content provider.[1] For example, a user may select AT&T as their ISP, and select Netflix and Hulu as their CPs. Generally, a user pays its ISP for bandwidth, and the CP may also pay the ISP to deliver its content to users. Furthermore, a CP's income could come from advertisement services, subscription fees the user pays them monthly, and so on. In our analysis, we compute each CP's income based on its consumed bandwidth units. For instance, if an average user consumes 10 GB of data per month from a given CP, and the CP makes \$10 per month on average from each user, the CP's per-bandwidth unit income would be \$1/GB. Such payments are the basis of a zero-rating service, based on which we develop our model.

The development of our model takes place through a sequence of steps. Whereas the details are in the subsections, the logical progression is as follows: In Section 2.1, we first generalize the set of CPs and ISPs to account for users that might choose multiple providers or none. We then formalize the notion of zero-rating in Section 2.2. Next, in Section 2.3, we define the complementary choice model followed by the user model in Section 2.4.

[1] Note that the ISP's side of the market can also be extended to a model where users have multiple ISPs at a time, similar to the way CP's side is modeled. However, since it is a realistic scenario where each customer has one ISP and multiple CPs, we set that as the basis of our model.

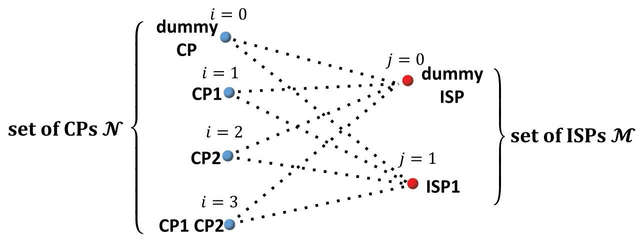


Fig. 1. The ISP-CP relations in a market of two real CPs and one real ISP, where we assume there also exist one dummy ISP, one dummy CP, and another CP bundle. Each user can choose one ISP $j \in \mathcal{M}$ and one CP $i \in \mathcal{N}$, and will be assigned to edge (i, j) accordingly.

2.1 Dummy ISP, Dummy CP, and CP Bundles

In some cases, there exist users who may not choose any ISPs or may not choose any CPs. For instance, users who select an ISP but no CP utilize Internet access but do not pay for premium services like Netflix or HBO. Users who select CPs but no ISP receive a premium service via another means, e.g., Netflix by DVD through mail. Users who select no ISP and no CP are those who require neither of these services. However, some users may utilize multiple CPs at a time.

To facilitate the analysis, rather than having users who select no ISP or no CP (or neither), we assume the existence of a *dummy* CP and a *dummy* ISP. Users who would select no ISP (CP) can be mapped to a setting where they select the dummy ISP (CP) at no cost. Note that the dummy ISP carries no traffic and the dummy CP has no content to offer. A user, therefore, has a choice of $|\mathcal{M}| = |\mathbb{M}| + 1$ ISPs, where $\mathcal{M}$ is the set of ISPs including the dummy one. However, since we assume that the users may choose multiple CPs at the same time, we define the concept of *CP bundles.* In this notion, we assume that there exist $|\mathcal{N}| = 2^{|\mathbb{N}|}$ possible subsets of CPs for the users to choose from, where $\mathcal{N}$ includes all combinations of CPs including the null set (the dummy CP). Some of these CPs reflect the actual CPs in the market, and some are auxiliary defined ones that are formed from possible combinations of the actual CPs. We refer to those as *CP bundles*, where the users can be uniquely assigned to a specific CP (bundles or actual) based on the content that they consume. Therefore, we denote the set of ISPs and CPs (including dummies and auxiliaries) by $\mathcal{M}$ and $\mathcal{N}$, respectively. Thus, a user always picks an ISP from $\mathcal{M}$ for their Internet access and a CP from $\mathcal{N}$ for the content.

Note that when the number of CPs increases, the computation time also increases exponentially. But we assume that the speed of simulation is not a concern in the theoretical findings, since this research does not entail real-time and speed-sensitive simulations but rather focuses on the derived conclusions. That said, in the real-world scenarios, not all combinations of CPs would have enough users and the baseline market shares of some CP bundles can be approximated by 0. Therefore, the matrix of Φ would become sparse as the market size grows, which would help with the running time of the simulations. In the current work, however, due to the confidentiality of real-world data, we have made an exhaustive analysis of the market parameters, where our conclusions are general and can be applied to real-world scenarios.

Figure 1 illustrates the case where the market originally has $|\mathbb{N}| = 2$ CPs and $|\mathbb{M}| = 1$ ISP. We model the market by including the dummy ISP, the dummy CP, and CP bundles, where we have $|\mathcal{N}| = 2^{|\mathbb{N}|} = 4$ and $|\mathcal{M}| = |\mathbb{M}| + 1 = 2$. Note that the relationship between CPs and ISPs is illustrated in a complete bipartite graph, where the users are assigned to different edges. More specifically, if a user accesses CP 1 through ISP 1, then they will be assigned to the edge connecting $i = 1$ to $j = 1$. If a user chooses ISP 1 as their internet provider and both CP 1 and CP 2, then they will be assigned to the edge connecting $i = 3$ to $j = 1$. Each user can be assigned to one and only one edge, therefore, the number of customers assigned to different edges should sum up to the total market size.

2.2 Zero-rating

In a paradigm where zero-rating is permitted, an ISP and a CP may agree that instead of the user, the CP will cover the *bandwidth* cost of the content viewed by the user, potentially at a price lower than what the user would pay. This allows the ISP and CP to advertise that when the user signs up with that ISP, they receive content from this particular CP for free. A user's choice of ISPs and CPs could depend upon whether zero-rating is permitted and offered by an ISP-CP pair as a service.

We assume customers are mapped to an ISP-CP pair (i, j) according to a pre-determined distribution. However, the distribution is affected by zero-rating choices; if ISP j and CP i zero-rate, the probability of customers being assigned to them changes. We denote the zero-rating relationship between CP $i \in \mathcal{N}$ and ISP $j \in \mathcal{M}$ by $\theta_{ij} \in \{0, 1\}$, where $\theta_{ij} = 1$ indicates zero-rating between i and j is established, otherwise $\theta_{ij} = 0$.

Even though zero-rating does not apply to the dummy CP $i = 0$ or ISP $j = 0$, we always assume $\theta_{0j} = \theta_{i0} = 1;\ \forall i \in \mathcal{N},\ j \in \mathcal{M}$. In dummy providers, zero-rating is an abstract term, and as we will see in Section 2.4, these providers are always assumed to have zero-ratings. Different from dummy providers, zero-rating relation can be extended to CP bundles. Since a CP bundle is a subset of actual CPs, it is assumed to zero-rate with a given ISP j if and only if all the CPs comprising it zero-rate with the ISP. For instance, in Figure 1 the CP bundle 3 is assumed to have a zero-rating relation with ISP 1 if and only if both CP 1 and CP 2 have a zero-rating relation with ISP 1, i.e., $\theta_{31} = 1 \iff \theta_{11} = 1\ \&\ \theta_{21} = 1$.

2.3 Complementary Choices Model $(\mathcal{N}, \mathcal{M})$

We denote the *baseline market share* of ISP j by $\psi_j \in (0, 1]$, which captures the *intrinsic* characteristics such as price and brand name, and models the market share of ISP j when none of the ISPs in the system zero-rate with the CPs. If a provider offers higher bandwidth and/or lower cost, i.e., a high bandwidth-to-cost ratio, then it would have a higher baseline market share than that of a provider with a lower bandwidth-to-cost ratio. In that case, the percentage ψ_j of each CP i's users will choose ISP j from the set $\mathcal{M}$ of ISPs. In probabilistic choice models [21], ψ_j can also be interpreted as the probability that any user chooses ISP j where none of ISPs offer zero-rating, and we have $\sum_{j \in \mathcal{M}} \psi_j = 1$. Furthermore, we denote the baseline market share of CP i by $\phi_i \in (0, 1]$, i.e., the market share of CP i when none of CPs zero-rate with the ISPs, and $\sum_{i \in \mathcal{N}} \phi_i = 1$.

We define $\boldsymbol{\psi} \triangleq (\psi_1, \ldots, \psi_{|\mathcal{M}|})^T$ and $\boldsymbol{\phi} \triangleq (\phi_1, \ldots, \phi_{|\mathcal{N}|})^T$. Similarly, we have $\boldsymbol{\theta_i} \triangleq (\theta_{i1}, \ldots, \theta_{iM})$ and $\boldsymbol{\vartheta_j} \triangleq (\theta_{1j}, \ldots, \theta_{Nj})$ as the zero-rating profile of CP i and ISP j, respectively. The zero-rating matrix of the whole system is defined as $\Theta \triangleq \{\theta_{ij} : i \in \mathcal{N}, j \in \mathcal{M}\}$. Given any zero-rating strategy Θ, we denote the strategies of all pairs of providers other than CP i and ISP j by Θ_{-ij}.

We also define α to be the fraction of elastic users of the market who choose among the CPs and ISPs with zero-rating relations, and if no such providers exist, these users would be distributed among all the providers. The rest of the users are distributed among CPs and ISPs merely based on their baseline market shares and independent of the zero-rating relations. These users are referred to as non-elastic, or sticky users, who comprise $1 - \alpha$ fraction of the market. Note that the sticky users do not move among the providers if their zero-rating relations change, while elastic users do.

In practice, users may choose services from constrained sets of CPs and ISPs, Table 1. It might be because certain providers are not available to the users or cannot satisfy their requirements. In general, we denote a set of choice pairs by $\mathcal{L}$. This set of available choices is impacted by zero-rating relations in the system, i.e., Θ. If CP i stops the zero-rating relationship with ISP j such that

Table 1. Summary Description of Parameters

Parameter	Description
(i, j)	a pair of CP i and ISP j
$\mathcal{N}, \mathcal{M}$	set of all CPs, ISPs (including dummy and bundles)
$\mathbb{N}, \mathbb{M}$	set of actual CPs, ISPs
θ_{ij}	zero-rating relation between (i, j)
X_{ij}	#users of (i, j); $i \in \mathcal{N}, j \in \mathcal{M}$
$\mathbb{X}_{ij}$	the effective #users of (i, j); $i \in \mathbb{N}, j \in \mathbb{M}$
α	user elasticity
c	bandwidth usage coefficient for non-zero-rated data
q_i	per bandwidth revenue of CP i; $i \in \mathbb{N}$
p_j	per bandwidth price of ISP j; $j \in \mathbb{M}$
ϕ_i	baseline market share of CP i; $i \in \mathcal{N}$
ψ_j	baseline market share of ISP j; $j \in \mathcal{M}$
δ_j	price discount of ISP j; $j \in \mathbb{M}$
U_i, R_j	utility of CP i, ISP j; $i \in \mathbb{N}, j \in \mathbb{M}$

the bandwidth costs for their services on that ISP is no longer zero, then their users might switch to an alternative ISP and/or CP in the market.

Assumption 1. *The set $\mathcal{L}$ for sticky users is the entire choice set $\mathcal{N} \times M$. The set $\mathcal{L}$ for elastic users is any pair of CP i and ISP j who zero-rate with one another.*

Assumption 1 states that elastic users of the market only choose among pairs of CP-ISP that zero-rate, since they want to maximize their surplus. However, since zero-rating does not impact the decision of the sticky users, those users are distributed among all ISPs and CPs based on the baseline market shares. In other words, they stay with their initial providers regardless of Θ.

Based on the baseline market shares of the providers, we make the following assumption on the users' choices.

Assumption 2. *Given a nonempty set $\mathcal{L}$ of available choices, a user chooses a choice pair $(i, j) \in \mathcal{L}$ with probability*

$$\mathbb{P}_{\mathcal{L}}\{(i,j)\} = \frac{\phi_i \psi_j}{\sum_{(n,m)\in\mathcal{L}} \phi_n \psi_m}. \tag{1}$$

Under Assumption 2, if $\mathcal{L}$ equals the choice set $\mathcal{N} \times M$, then the probability of choosing (i, j) equals $\phi_i \psi_j$, which is consistent with our notion of *baseline market shares*. Furthermore, by Luce's choice axiom [22], the proportional form in Equation (1) is also necessary for guaranteeing an *independence from irrelevant alternatives* (IIA) property: the probability of selecting one item over another from a pool of many items is not affected by the presence or absence of other items in the pool.

2.4 User Model

Our complementary choices model $(\mathcal{N}, M)$ can be specified by a triple of vectors $(\boldsymbol{\phi}, \boldsymbol{\psi}, \Theta)$ and the scalar α. We denote the total market size by X. Based on Assumptions 1 and 2, we characterize the number of users of (i, j), denoted by X_{ij}, as a function of the zero-rating matrix Θ of the system as $X_{ij}(\Theta) = \rho_{ij}(\Theta)X$, where ρ_{ij} is the closed form market share of the pair (i, j) and we have

$$\rho_{ij}(\Theta) = \left[\alpha \times \frac{\phi_i \psi_j \theta_{ij}}{\sum_{i'}\sum_{j'} \phi_{i'} \psi_{j'} \theta_{i'j'}} + (1-\alpha) \times \phi_i \psi_j\right] \mathbf{1}_{\{\Theta \neq \mathbf{0}\}} + \left[\phi_i \psi_j\right] \mathbf{1}_{\{\Theta = \mathbf{0}\}}. \tag{2}$$

Equation (2) derives the number of users X_{ij} for any pair (i, j) of complementary providers under the zero-rating profile Θ. In particular, X_{ij} can be represented by the number of users in the system

X multiplied by the closed-form market share $\rho_{ij}(\Theta)$, which is a function of the zero-rating profile Θ, the baseline market shares, and the elasticity of the users. If CP i and ISP j do not zero-rate, i.e., $\theta_{ij} = 0$, then the pair (i, j) of providers keep the proportion $1-\alpha$ of sticky users who are distributed in the system based on baseline market shares. However, if CP i zero-rates with ISP j, i.e., $\theta_{ij} = 1$, then not only they will keep the proportion $1 - \alpha$ of their sticky users but also the elastic users in the market would have incentives to choose them. Since elastic users are distributed among the providers who offer zero-rating, α fraction of users would choose this pair with probability proportional to their baseline market shares. If none of the CPs and ISPs offer zero-rating, then all users are again distributed among the providers based on their baseline market shares.

Note that a popular $4k$ streaming CP, like Netflix, would have a higher ϕ_i than a low-resolution counterpart. This will in turn increase the market share of such high-resolution and bandwidth-demanding CP in Equation (2). Since the elasticity of users is also multiplied by a function of ϕ_i in that Equation, CPs like Netflix would attract a higher number of elastic users when they offer zero-rating. Therefore, this will capture how users value zero-rating differently for different CPs.

As mentioned earlier, the dummy providers are never assumed to have zero-ratings. However, they can capture some elastic users of the market in case no CP zero-rates, as well as their own sticky users. Furthermore, as baseline market shares model intrinsic characteristics of a provider, a famous content provider such as Netflix or an ISP with low p_j would have higher baseline market shares than their competitors and as a result, they would have higher X_{ij}.[2]

Lemma 2.1. *Let $\mathcal{N}' \subseteq \mathcal{N}$ and $\mathcal{M}' \subseteq \mathcal{M}$. For any $n \notin \mathcal{N}$ and $m \notin \mathcal{M}$, let $\widetilde{\mathcal{N}} \triangleq \mathcal{N} \backslash \mathcal{N}' \cup \{n\}$ and $\widetilde{\mathcal{M}} \triangleq \mathcal{M} \backslash \mathcal{M}' \cup \{m\}$ denote the new sets of providers where the subsets $\mathcal{N}'$ and $\mathcal{M}'$ are replaced by the providers n and m, respectively. Let $\widetilde{X}_{ij}$ denote the number of users of (i, j) under $(\widetilde{\mathcal{N}}, \widetilde{\mathcal{M}})$. If $\boldsymbol{\theta_i} = \boldsymbol{\theta_n}$, $\forall i \in \mathcal{N}'$, $\boldsymbol{\vartheta_j} = \boldsymbol{\vartheta_m}$, $\forall j \in \mathcal{M}'$, and $(\phi_n, \psi_m) = (\sum_{i \in \mathcal{N}'} \phi_i, \sum_{j \in \mathcal{M}'} \psi_j)$, then*

$$\widetilde{X}_{nm} = \sum_{i \in \mathcal{N}'} \sum_{j \in \mathcal{M}'} X_{ij}; \quad \widetilde{X}_{ij} = X_{ij}, \ \forall i \neq n, j \neq m;$$

$$\widetilde{X}_{nj} = \sum_{i \in \mathcal{N}'} X_{ij}, \ \forall j \neq m; \ \textit{and} \ \widetilde{X}_{im} = \sum_{j \in \mathcal{M}'} X_{ij}, \ \forall i \neq n.$$

Lemma 2.1 states that if there exists multiple CPs (or ISPs) that use the same zero-rating profile, then they could be conceptually merged as a single CP (or ISP) without affecting the market shares of other providers.

When the users choose multiple CPs at the same time, they contribute to the revenues of all CPs they use, as well as the ISP they are assigned to. Let us assume that the set of CPs who are comprised from the actual CP i is shown by $AUX(i)$. For instance, in Figure 1, we have: $AUX(1) = \{1, 3\}$, and $AUX(2) = \{2, 3\}$. Therefore, the effective number of users who utilize the pair of and actual CP i

[2] The reader may note that in this model, if there is no zero-rating in the market and an elastic user is using CP1-CP2, after only one of the CPs, say CP2, zero rates, then the elastic user is only going to use CP2, but a higher portion of its data is consumed (based on Assumption 4, the consumption rate is increased by the coefficient of $1/c$, where $0 < c \leq 1$). Since we suggest that CPs under study have similar nature of content, this assumption makes sense. For instance, suppose there is an elastic user who uses both Netflix and HBO through Verizon, but since her data plan is limited, the user streams a small amount of data. If HBO's content becomes zero-rated through Verizon, then the elastic user increases the amount of streaming over HBO by $1/c$, since the user is not paying for that data anymore. Therefore, the streaming needs of the user are met and the elastic user would no longer need to use Netflix, which in turn becomes the more expensive option to stream from. Even though the content of Netflix and HBO are not 100% exchangeable, the elastic user is looking to maximize its consumer surplus and opts to limit herself to the cheaper content as long as it is of the same nature. Note that this does not hold for sticky users, who will stay with their initial providers because of the content and regardless of this zero-rating change in the market.

and ISP j can be computed from Equation (3):

$$\mathbb{X}_{ij} = \sum_{i \in AUX(i)} X_{ij}. \tag{3}$$

Corollary 2.2. *When all the providers $\mathcal{N} \times \mathcal{M} - \{(i,j)\}$ have fixed strategies, i.e., Θ_{-ij} is fixed, zero-rating (i,j), i.e., changing θ_{ij} from 0 to 1, always helps CP i to attract more customers.*

Based on Equation (2), in case CP i zero-rates with ISP j, since $\theta_{ij} = 1$, as the first term of the equation is a positive value, it causes X_{ij} to increase, which then increases $\mathbb{X}_{ij}$ based on Equation (3).

3 UTILITY AND ZERO-RATING EQUILIBRIA

Pricing takes various forms for the Internet in practice. Wireline ISPs often charge flat rates [23], while tiered schemes have recently been adopted by major U.S. broadband providers such as Verizon [24] and AT&T [25]. Although data pricing may take various forms, we assume each ISP, by choosing its pricing structure, decides along with each CP whether to adopt zero-rating for their customers.

While the data prices and values are assumed to be exogenous, each ISP j has the option of charging CPs a different data price $\delta_j p_j$ if they zero-rate, where $0 < \delta_j \leq 1$ denotes the data price discount ISP j offers to the CPs. If $\delta_j < 1$, then CPs can purchase ISP j's bandwidth as a zero-rated service, with a lower price than the users can directly purchase it. Each ISP j should strategically choose this price discount, since for some δ_j its total revenue could increase since it could attract a higher number of customers, even though its bandwidth unit income decreases. However, a lower value of δ_j could harm ISP j's total revenue. We define $\boldsymbol{\delta} \triangleq (\delta_1, \ldots, \delta_{|\mathcal{M}|})^T$ to denote the entire ISP discount profile of the market.

In this section, we analyze ISPs' and CPs' zero-rating decisions. Note that although the CP bundles do not make independent zero-rating decisions, we account for their users in our evaluations, since they contribute to the users of actual providers (Equation (3)), and therefore to their utilities. However, the users of dummy providers do not generate any utilities. We first introduce the following assumption to compute the utility model of the actual providers in the market.

Assumption 3. *The revenue of ISP $j \in \mathbb{M}$ from the market of CPs is equal to the summation of revenues each CP i's user brings to j for all $i \in \mathbb{N}$. Likewise, the utility of each actual CP $i \in \mathbb{N}$ is equal to the summation of utility each ISP j's user brings to i for all $j \in \mathbb{M}$.*

Note that while the utility is a general term, it can model the benefit a player gains in an abstract form. Since unlike ISPs, each CP's income has an indirect relationship with the bandwidth usage, we use the term *utility* to model its decision-making process. Whereas to avoid confusion, we use the term *revenue* to address the same thing for ISPs.

Assumption 4. *When zero-rating is provided for a pair of CP i and ISP j, i.e., $\theta_{ij} = 1$, since users would not pay the bandwidth price, their average bandwidth usage increases by a factor of $1/c$, where $0 < c \leq 1$.*

When the bandwidth usage increases in case of zero-rating, the utilities of CPs and the revenues of ISPs who zero-rate will also increase, since they are a function of per-bandwidth unit prices. To model this, instead of assuming the utilities and revenues of providers who zero-rate increase by a factor $1/c \geq 1$, for simplicity, we assume if the providers cancel their zero-rating, then their utilities and revenues decrease by a factor of $0 \leq c \leq 1$, which we call *bandwidth usage coefficient.* Note that our model is not designed to capture bandwidth saturation for the ISPs, assuming ISPs to be smart agents with mechanisms to provide the bandwidth requested by users, and in case of zero-rating the CPs will pay for ISPs' bandwidth. Furthermore, for simplicity, the ISPs and CPs aim to maximize their revenues without directly considering the bandwidth cost. We may assume this cost is indirectly modeled into p_i and q_j, which are the units for income for CP i and ISP j, respectively.

Using Assumptions 3 and 4, and given any zero-rating strategy profile Θ, we denote the utility of CP i by $U_i(\Theta)$ and the revenue of ISP j by $R_j(\Theta)$ and define them as

$$\begin{aligned} U_i(\Theta) &\triangleq \sum_{j\in\mathbb{M}} U_i^j(\Theta) \quad \text{and} \quad R_j(\Theta) \triangleq \sum_{i\in\mathbb{N}} R_j^i(\Theta), \\ \text{where} \quad U_i^j(\Theta) &\triangleq \begin{cases} cq_i\mathbb{X}_{ij}(\Theta) & \text{if} \quad \theta_{ij} = 0, \\ (q_i - \delta_j p_j)\mathbb{X}_{ij}(\Theta) & \text{if} \quad \theta_{ij} = 1, \end{cases} \\ \text{and} \quad R_j^i(\Theta) &\triangleq \begin{cases} cp_j\mathbb{X}_{ij}(\Theta) & \text{if} \quad \theta_{ij} = 0, \\ \delta_j p_j\mathbb{X}_{ij}(\Theta) & \text{if} \quad \theta_{ij} = 1. \end{cases} \end{aligned} \tag{4}$$

Each CP i's utility is the sum over the utilities U_i^j generated from each ISP j, which equals to the effective number of users $\mathbb{X}_{ij}(\Theta)$ multiplied by either its profit margin $q_i - \delta_j p_j$ if zero-rating, or its original value q_i otherwise. Similarly, each ISP j's revenue is the sum over the revenues R_j^i generated from each CP i, which equals the effective number of users $\mathbb{X}_{ij}(\Theta)$ multiplied by either the discounted price $\delta_j p_j$ if zero-rating, or by $p_j.c$ otherwise. Note that the utility functions of CP bundles are not well-defined, since we count their users as effective users of the actual CPs they encompass. Furthermore, the users of dummy providers do not generate any utility.

CPs' zero-rating decisions depend on the prices imposed by ISPs, and ISPs' decisions depend on the revenue they receive from CPs via zero-rating compared to what they would receive from users directly. Equations (2) and (3) show that CP i's utility $U_i(\Theta)$ depends not only on its own strategy θ_{ij} but also on all other CPs' and ISPs' strategies Θ_{-ij}. Given the price profile $\boldsymbol{p}$, ISPs make simultaneous zero-rating offers with deciding a discount profile $\boldsymbol{\delta}$ to maximize their revenues, and CPs make simultaneous decisions whether or not to adopt them. We define a zero-rating equilibrium as follows:

Definition 3.1 (Zero-rating Equilibrium). In a market of ISPs and CPs, given fixed discount and price profiles, a zero-rating strategy profile is a ***zero-rating equilibrium*** **(ZRE)** if and only if (1) given a zero-rating strategy Θ chosen by ISPs, neither of CPs would gain by unilaterally deviating from Θ, (2) given a zero-rating strategy Θ chosen by CPs, neither of ISPs would gain by unilaterally deviating from Θ.[3]

Based on Definition 3.1, if Θ is a ZRE, for each actual CP i and ISP j, then we have $U_i(\theta_{ij};\Theta_{-ij}) \geq U_i(\bar{\theta}_{ij};\Theta_{-ij})$ and $R_j(\theta_{ij};\Theta_{-ij}) \geq R_j(\bar{\theta}_{ij};\Theta_{-ij})$.

ZRE is a specific kind of Nash equilibrium [26], where there exist two groups of inter-dependent players. Since zero-rating is a bilateral contract between ISPs and CPs, the zero-rating decision that is affected by the entire market resembles a bargaining problem. For instance, given a pair of ISP-CP, the CP (ISP) does not have the option of zero-rating if the ISP (CP) is not willing to zero-rate. Therefore, we use the term ZRE to avoid confusion. ZRE is evaluated for a *pure strategy* game, since

[3]Note that ZRE based on discount profile, i.e., $\boldsymbol{\delta}$, is a two-stage game. Even though the definition of ZRE (Definition 3.1) assumes that $\boldsymbol{\delta}$ is fixed, it reflects the final stage of the game and the results hold even if $\boldsymbol{\delta}$ is not fixed. In other words, let us say the game is played multiple times to reach an equilibrium: $\boldsymbol{\delta}$ is updated, then the ZRE profile is updated, then $\boldsymbol{\delta}$ is updated again, and so on until everything is stable. Eventually, $\boldsymbol{\delta}$ will be fixed in the final stage before reaching ZRE. Hence, the theoretical results will hold.

mixed strategy decisions between CP i and ISP j to zero-rate do not apply to real-world scenarios, and users need deterministic knowledge on which ISP-CPs offer zero-rating.

In some ISP prices, although ZRE is where a CP i zero-rates, it may face a utility drop compared to the case where no zero-rating is allowed in the market, i.e., $\Theta = \mathbf{0}$. In this case, if CP i deviates, it loses customers to the CPs who zero-rate, and its utility further drops. This scenario resembles *prisoner's dilemma* paradox in Reference [27], where each player chooses to protect themselves at the expense of the other participant and as a result, the optimal outcome will not be produced. However, since the market of CPs is mostly heterogeneous, this scenario mainly harms the low-value CPs rather than high-value ones, as we see in Section 4.2. We define *zero-rating pressure* to address this phenomenon.

Definition 3.2 (Zero-rating Pressure). Zero-rating pressure happens when a CP decides to zero-rate to avoid losing customers; only because some other CPs are offering zero-rating, and if no other CPs zero-rate, the former would not zero-rate either.

Suppose there are two heterogeneous CPs in the market, and in ZRE, the CP with a lower value zero-rates with an ISP. In that case, its utility shall improve compared to when it does not zero-rate. If zero-rating increases low-value CP's utility, then the same must be true for the high-value CP as well. We introduce Lemma 3.3 to generalize this case.

LEMMA 3.3. *In a market $\mathbb{N}$ of content providers with different values, suppose $i, i' \in \mathbb{N}$ and $q_i < q_{i'}$. The zero-rating strategy that CP i zero-rates with ISP j while CP i' does not is never a ZRE.*

A more rigorous proof of Lemma 3.3 is provided in Appendix A, which will be used in the next section to analyze how zero-rating impacts the market.

4 ANALYSIS

In this section, we represent a macroscopic and microscopic analysis of the impact of zero-rating on CPs. For the former, we look at the CPs as a whole and evaluate the Herfindahl index [19] of the market, which is a proxy of competitiveness. For the latter, we look into individual CPs' utilities.

4.1 Herfindahl Index Analysis

To show the impact of zero-rating on the market, we compute the *Herfindahl index* [19] among CPs, also known as *Herfindahl-Hirschman index* or *HHI*. This index is calculated as the sum of squares over the market shares of all firms in the market. Since it accounts for the number of firms and concentration, it is generally used as a proxy of competitiveness [19]. When this index grows to 1, the market moves from a competitive state to a monopolistic content provider, i.e., the competition decreases. Lack of competition in the market causes market distortion and significant welfare loss due to monopoly [28]. HHI increases of over 0.01 generally provoke scrutiny, although this varies from case to case [29]. Herfindahl index is also usually used to measure the impact of a change on market power,[4] as its increase generally indicates a decrease in market power and vice versa [31].

In this section, we analyze a market of CPs with different values and show how the availability of zero-rating impacts the Herfindahl index of the market. The analysis of this section is based on the user model in Equation (2), and since the conclusions are theoretical, they are general to our model and are independent of parameter choices. The detailed proofs of the lemmas in this section are present in Appendix A.

[4] Market power refers to the ability of a firm to manipulate the price of a product in the market to increase economic profit [30].

Lemma 4.1. *Let the market share of content provider i after the equilibrium be X_i/X. In the market $\mathbb{N}$ of content providers, the Herfindahl index increases when the variance of content providers' market shares, i.e., the variance of $X_i/X : i \in \mathbb{N}$, increases.*

Based on Lemma 4.1, the more different the market shares of CPs are, the higher the Herfindahl index would be. Intuitively, a high variance between the market shares indicates that the market is moving towards a monopoly, where the increase in the Herfindahl index confirms that as well.

Lemma 4.2. *The Herfindahl index is the same if none of the CPs zero-rate versus if every CP in the market zero-rates.*

The amount of consumption may increase in case every CP zero-rates in the market compared to when no one zero-rates. However, since the relative market share of CPs remains unchanged (see Appendix A for proof), the Herfindahl index stays the same in both cases.

Theorem 4.3. *In a market $\mathbb{N}$ of content providers with values $\{q_1, q_2, \ldots, q_{|\mathbb{N}|}\}$, suppose $q_1 \leq q_2 \leq \ldots \leq q_{|\mathbb{N}|}$, and the content providers with higher values also have higher baseline market shares, i.e., $\phi_1 \leq \phi_2 \leq \cdots \leq \phi_{|\mathbb{N}|}$. If at least one of these inequalities is strict, then zero-rating options in the market will cause the Herfindahl index to be non-decreasing in all possible ZRE.*

Given Lemma 4.2, when zero-rating is available in the market, in case ZRE consists of either every CP or no CP zero-rate, the Herfindahl index stays the same. In other ZRE cases, based on Theorem 4.3 if CPs with higher values and higher baseline market shares afford more zero-ratings than their low-value opponents, the Herfindahl index will increase (proof in Appendix A). Note that at least one of the inequalities needs to be strict, since otherwise if they are all equalities, the content providers would behave the same in the equilibria and can be conceptually merged. This theorem states a realistic scenario, as the popular content providers usually have both higher per bandwidth unit income (value), and higher number of customers (market share) than the smaller startups. Therefore, this Theorem could represent the case where startups with low incomes and low initial baseline market shares join the market of CPs. The increase in the Herfindahl index implies that the market moves toward monopoly, where the startups would not survive.

4.2 Utility Analysis

Computing the utility for each content provider requires prior knowledge of the ZRE strategies. Note that for a two-player game (or more), neither existence nor uniqueness of Nash equilibria could be guaranteed; it is $\mathcal{NP}$-complete to determine whether the Nash equilibria with certain natural properties exist [32] and it is $\#\mathcal{P}$-hard to count the Nash equilibria [33]. However, in a heterogeneous market of CPs with different values, based on Lemma 3.3, there are a limited number of zero-rating strategies that could become an equilibrium, i.e., the case where a low-value CP zero-rates with an ISP, while another CP with higher value does not, is never an equilibrium while the opposite can be. Therefore, we focus on how zero-rating impacts CPs with different values, where incumbents and startups could be thought of as CPs with high and low values, respectively. We introduce Theorem 4.4 to address the special case where the low-value CP cannot afford to zero-rate in the equilibria, while the high-value CP can.

Theorem 4.4. *In a market of CPs with $|\mathbb{N}| \geq 2$, let CP i have a lower value than CP i'. If the low-value CP i does not zero-rate with any ISP ($\theta_{ij} = 0\ \forall j \in \mathbb{M}$), while the high-value CP i' does ($\theta_{i'j} = 1\ \exists j \in \mathbb{M}$), and the zero-rating strategy profile for the rest of CPs (other than CP i and CP i') does not change, then the utility of low-value CP i will always decrease compared with when zero-rating is not available, while the utility of high-value CP i' increases or remains the same.*

Since utility analysis, in general, depends on the exact zero-rating strategies of the market after equilibria, which are not possible to be determined in a closed-form formula, we perform numerical evaluations to illustrate the impact of zero-rating on the CPs in next section.

5 EVALUATION AND RESULTS

In this section, we analyze the zero-rating equilibria for a market with complementary duopoly, i.e., $|\mathbb{M}| = |\mathbb{N}| = 2$, where two CPs and two ISPs compete on both sides of the market. Assuming the regulation of *weak content provider net neutrality* [4], where each ISP charges the same price from every CP. For the simplicity of the result illustration, we also assume that ISPs' price discount profile $\boldsymbol{\delta} = \mathbf{1}$. Therefore, the ISPs charge the same price from the CPs in case of zero-rating as they would charge the users otherwise. We have separately analyzed the case where ISPs get to decide on the discount profile $\boldsymbol{\delta}$ in Appendix B and found that the results are qualitatively the same.[5]

As Lemma 2.1 shows that providers with similar prices and zero-rating strategies can be merged, a duopolistic model provides a first-order approximation of market competitions from a provider's perspective where all its competitors are considered as an aggregated provider that captures the remaining market share. We assume the elasticity of the users, baseline market shares, and prices are exogenous. Note that this evaluation can be extended to different parameter choices and is not prone to parameter selection. However, to determine these parameters in a real-world market, the reader may refer to some previous work that study the impact of zero-rating on mobile Internet usage and have been reviewed in Section 6. We use a general model and synthetically set the parameters to show how the market behaves under different conditions, and that parameter selection **does not qualitatively change** our results. All prices and revenues are normalized to 1 and are not intended to reflect *absolute* real-world values, rather the relative differences between ISPs and CPs.

We compare two different hypothetical markets, one where zero-rating is not allowed, the other one where zero-rating is allowed, and ISPs and CPs decide on their zero-rating strategies where the market could reach the equilibria. If **multiple ZRE** exist, to focus on the impact of zero-rating, then we make the choice to illustrate only one ZRE with the maximum number of zero-rated relations, and in case of a tie, the high-value CP (in our case CP 2) and ISP 2 will have the maximum number of zero-ratings. If **no ZRE** exists, then we assume the market-share and utility of CPs remain the same as if there are no zero-rating options in the market. We show how the zero-rating strategies are made and how HHI and the utilities of CPs change in these two markets.

5.1 Benchmark Scenario

We use a benchmark scenario in which $c = 0.5$, $\alpha = 0.5$, and $\boldsymbol{\delta} = \mathbf{1}$. We also have $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, and $\boldsymbol{\psi} = (0.2, 0.4, 0.4)$, where assuming the vector indices start from 0, ϕ_0 is the baseline market share of dummy CP, ϕ_3 is the baseline fraction of customers who use *both* CP 1 and CP 2, and ϕ_1 and ϕ_2 are the baseline market shares of CP 1 and CP 2, respectively. Similarly, ψ_0 is the baseline market share of dummy ISP, and ψ_1, and ψ_2 are the baseline market shares of ISP 1 and ISP 2, respectively. Without loss of generality, we assume $q_1 \leq q_2$ and normalize the prices such that $\boldsymbol{q} \leq \mathbf{1}$ and $\boldsymbol{p} \leq \mathbf{1}$.

[5]Including the decision on $\boldsymbol{\delta}$ changes the model to a bargaining problem on an extra dimension on $\boldsymbol{\delta}$. Furthermore, the number of Nash Equilibria increases as well for each price profile (p_1, p_2), and choosing one to depict in the graph makes it harder to grasp for the reviewer at first glance. Therefore, for simplicity, we assume that $\boldsymbol{\delta}$ is fixed, but in Appendix B, we show how our model is able to be generalized to the choice of ISPs on $\boldsymbol{\delta}$ and how it can successfully capture it. Furthermore, our assumptions and conclusions are valid when ISPs make decisions on the discount profile $\boldsymbol{\delta}$ as well.

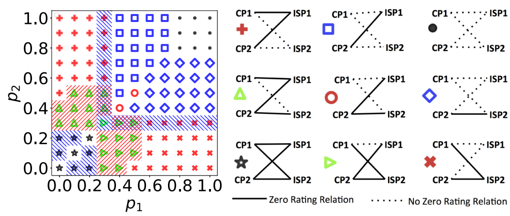
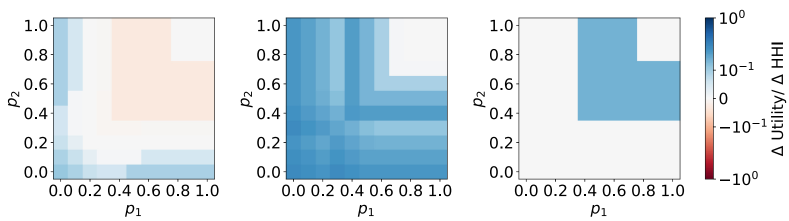


Fig. 2. ZRE map under complementary duopoly with $\alpha = 0.5$, $c = 0.5$, $\boldsymbol{\delta} = (1.0, 1.0)$, $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, $\boldsymbol{\psi} = (0.2, 0.4, 0.4)$, and $\boldsymbol{q} = (0.4, 1.0)$. Shaded areas in blue (\) and red (/) represent zero-rating pressure for CP 1 and CP 2, respectively.



(a) CP 1's utility changes (b) CP 2's utility changes (c) HHI changes

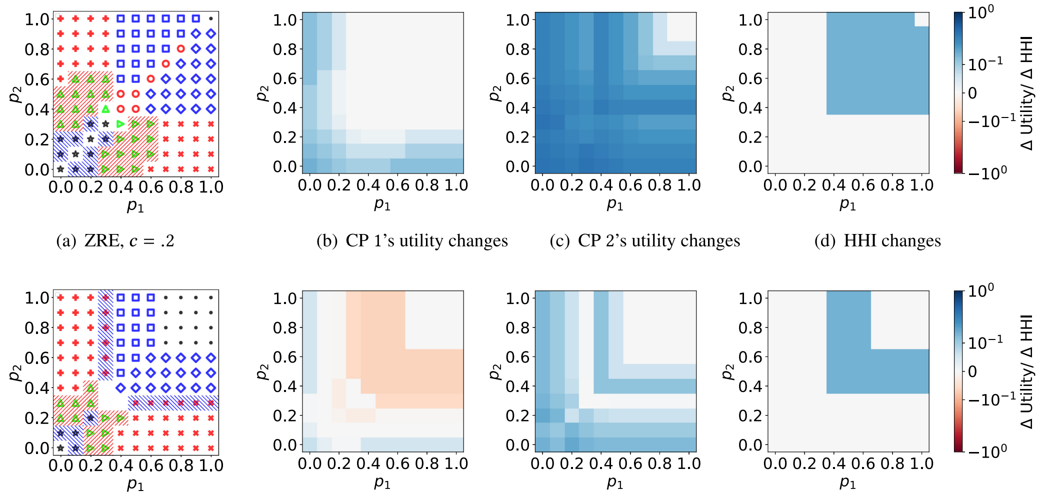
Fig. 3. The differences in CPs' utilities and HHI when zero-rating is available and the market reaches equilibria, minus when zero-rating is not available. We have: $\alpha = 0.5$, $c = 0.5$, $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, $\boldsymbol{\psi} = (0.2, 0.4, 0.4)$, $\boldsymbol{\delta} = (1.0, 1.0)$, and $\boldsymbol{q} = (0.4, 1.0)$.

Figure 2 visualizes the ZRE when ISPs' prices p_1 and p_2 vary along the x- and y-axis, respectively. Based on Lemma 3.3 as $q_2 > q_1$, 9 of the 16 possible zero-rating profiles could become ZRE under various ISP prices, which are shown in the right sub-figure as legends. The price of 0 of ISP j can represent when it offers an unlimited plan, and we assume in that case it is always zero-rating with all the CPs. Intuitively, when ISP prices are low, both CPs are willing to zero-rate; but when the prices are high, neither CP is willing to do so. Low-value CP 1 is generally more susceptible to ISP price changes; we observe that under any fixed price $p_{\bar{j}}$ as p_j increases, CP 1 first cancels its zero-rating relations with ISP j, followed by CP 2.

Figure 2 also illustrates the regions of zero-rating pressure for CPs. The shaded blue regions demonstrate when CP 1 zero-rates under pressure, and the shaded red regions demonstrate when CP 2 does so. When the low-value CP zero-rates with the cheaper ISP, the high-value CP as a response may zero-rate with the more expensive ISP to maintain its customers, which represents zero-rating pressure for it. However, any zero-rating of the high-value CP can cause pressure for the low-value CP, if it is not originally willing to zero-rate.

Figure 3 visualizes the impact of zero-rating on the CPs' utilities and HHI. We observe that in an imbalanced market of CPs, zero-rating *usually* decreases the utility of low-value CP 1, but increases the utility of high-value CP 2. Based on Lemma 3.3 (and Figure 2), high-value CP 2 always can afford more zero-ratings than low-value CP 1. Hence, its utility mostly increases as it attracts more elastic users of the market. This figure also confirms Theorem 4.4, where in case CP 1 does not zero-rate while CP 2 does, only CP 1's utility decreases. Figure 3(c) also shows how HHI is always non-decreasing after ZRE as opposed to when it is not allowed, which confirms Theorem 4.3.

In what follows, we focus on CPs side of the market and show how each parameter impacts ZRE strategies, CPs' utilities, and HHI.



(a) ZRE, $c = .2$ (b) CP 1's utility changes (c) CP 2's utility changes (d) HHI changes

(e) ZRE, $c = .8$ (f) CP 1's utility changes (g) CP 2's utility changes (h) HHI changes

Fig. 4. ZRE under complementary duopoly (a, e). The differences in CPs' utilities and HHI when zero-rating is available and the market reaches equilibria, minus when zero-rating is not available (b, c, d, f, g, h). We have $\alpha = 0.5$, $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, $\boldsymbol{\psi} = (0.2, 0.4, 0.4)$, $\boldsymbol{\delta} = (1.0, 1.0)$, and $\boldsymbol{q} = (0.4, 1.0)$.

5.2 Impact of Bandwidth Usage Coefficient

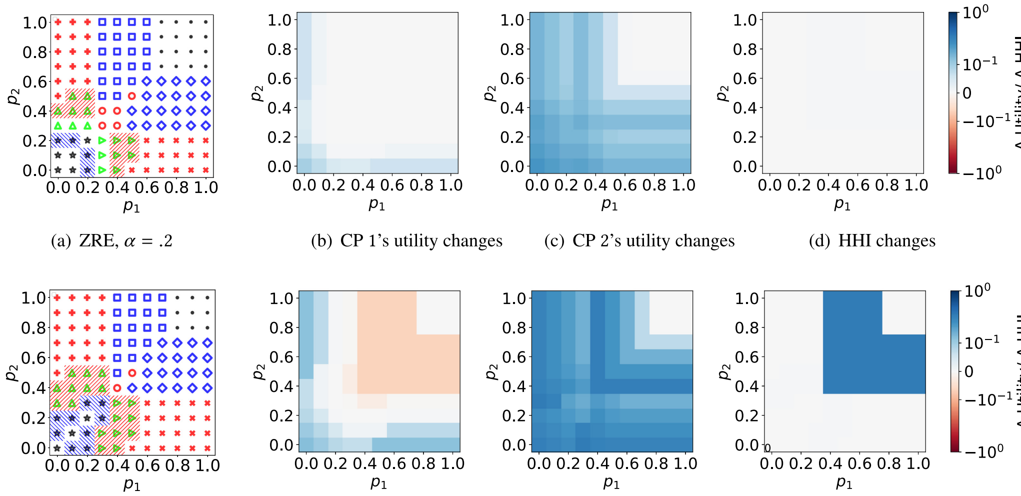
To show the impact of bandwidth usage on the market, we vary the parameter c and observe the equilibria, utilities, and HHI. Since our benchmark analysis in Figure 2 has $c = 0.5$, we test our model with both a lower and a higher value of c.

Figures 4(a) and 4(e) illustrate the impact of c on ZRE. When c decreases from 0.5 in Figure 2 to 0.2 in Figure 4(a), the number of zero-ratings increase, since canceling zero-rating causes less bandwidth usage, which produces less income for both CPs and ISPs. The regions of zero-rating pressure for low-value CP 1 also decreases as CP 1's incentives to zero-rate increase, and when it does so, it pressures CP 2 to increase its zero-rating regions as well. However, when $c = 0.8$ in Figure 4(e), canceling zero-rating would not decrease the bandwidth usage as much as it does in Figure 2. Hence, the number of zero-ratings decreases.

Figure 4 also depicts the utility and HHI changes corresponding to Figures 4(a) and 4(e). When $c = 0.2$, both CPs may experience an increase in their utilities, which is more major for high-value CP 2. Note that in regions where CP 2 is the only one who zero-rates, as shown by Theorem 4.4, CP 1's utility decreases, although it is not observable in the graphs. However, when $c = 0.8$, low-value CP 1 experiences a major utility loss, whereas the utility of CP 2 mostly increases. Finally, HHI in both cases increases as expected by Theorem 4.3.

Note that when CP 1 does not zero-rate but CP 2 does, the utility difference between ZRE and when no zero-rating is allowed is smaller for CP 1 when $c = 0.8$ compared to the benchmark ($c = 0.5$) and $c = 0.2$. When only CP 2 zero-rates, for all values of c the utility difference for CP 1 is negative. However, based on Equation (4), since the utility of CP 1 with no zero-ratings is multiplied by c, the larger c is, the smaller this negative number would be after multiplication.

Takeaways: When the bandwidth usage coefficient (c) is small, low-value CP sometimes and high-value CP always experience utility gain after zero-rating; this gain is higher for the high-value CP. When c is in the mid-range ($c = 0.5$), low-value CP primarily experiences utility loss, while high-value CP experiences utility gain. For high values of c ($c = 0.8$), the value of utility loss for low-value CP is more major, while the utility gain for high-value CP is smaller.



(a) ZRE, $\alpha = .2$ (b) CP 1's utility changes (c) CP 2's utility changes (d) HHI changes

(e) ZRE, $\alpha = .8$ (f) CP 1's utility changes (g) CP 2's utility changes (h) HHI changes

Fig. 5. ZRE under complementary duopoly (a, e). The differences in CPs' utilities and HHI when zero-rating is available and the market reaches equilibria, minus when zero-rating is not available (b, c, d, f, g, h). We have $c = 0.5$, $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, $\boldsymbol{\psi} = (0.2, 0.4, 0.4)$, $\boldsymbol{\delta} = (1.0, 1.0)$, and $\boldsymbol{q} = (0.4, 1.0)$.

5.3 Impact of User Elasticity

To observe how the elasticity of the users impacts the market decisions, we have simulated the market equilibria for $\alpha = 0.2$ and $\alpha = 0.8$, which are lower and higher than the benchmark $\alpha = 0.5$, respectively.

Figures 5(a) and 5(e) depict the impact of α on ZRE. As the user elasticity increases, the providers are more willing to zero-rate, since they can attract the elastic users of the market and increase their utilities. The regions of zero-rating pressure also increase with the user elasticity, because in some ISP prices where only one CP is willing to zero-rate, if the other CP does not it would lose more customers due to high elasticity.

Figure 5 also depicts the utility and HHI changes corresponding to Figures 5(a) and 5(e). Since CP 1 is less likely to afford zero-rating compared to CP 2, with higher user elasticity, high-value CP 2 could attract elastic users if it is the only one who zero-rates. Therefore, low-value CP 1 mostly has utility loss as α increases. As before, HHI remains increasing with is an indication of the market moving towards a monopoly. While the increase is not observable in the graph for $\alpha = 0.2$, it is much more major for $\alpha = 0.8$.

Takeaways: When user elasticity increases, it mostly harms low-value CP in ZRE, since the higher fraction of its users will be elastic and leave it when it does not afford zero-rating.

5.4 Impact of Baseline Market Shares

To show the impact of baseline market shares, $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$, we vary the values of ϕ_1 and ϕ_2 for CP 1 and CP 2, and ψ_1 and ψ_2 for ISP 1 and ISP 2. In our benchmark simulation in Figures 2 and 3, we showed how ZRE, utilities and HHI look when $\phi_1 = \phi_2$ and $\psi_1 = \psi_2$. In this section, we evaluate our model for $\phi_1 > \phi_2$, $\phi_1 < \phi_2$, $\psi_1 > \psi_2$, and $\psi_1 < \psi_2$.

Figures 6(a) and 6(e) illustrates the impact of CPs' baseline market shares on the equilibria. We observe that when the baseline market share of the low-value (high-value) CP increases (decreases),

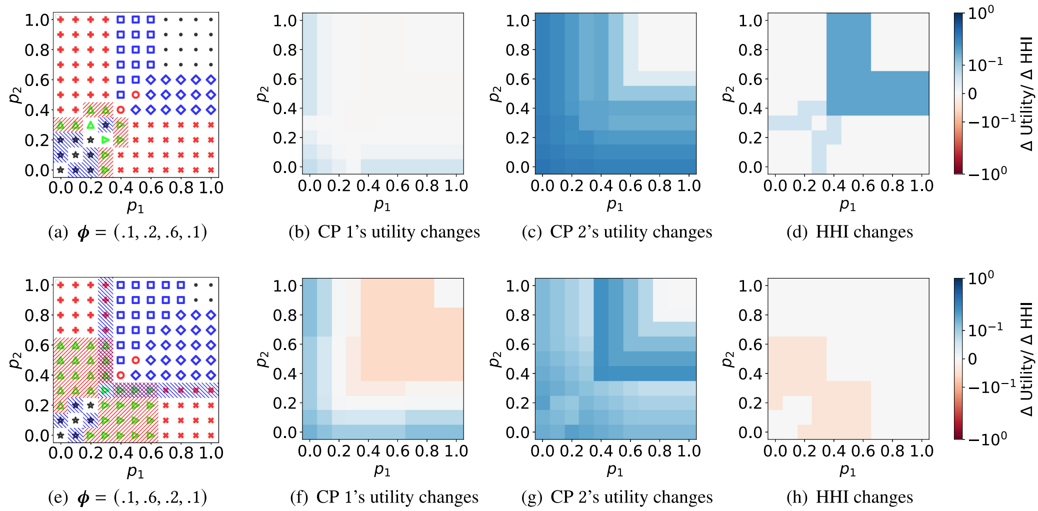


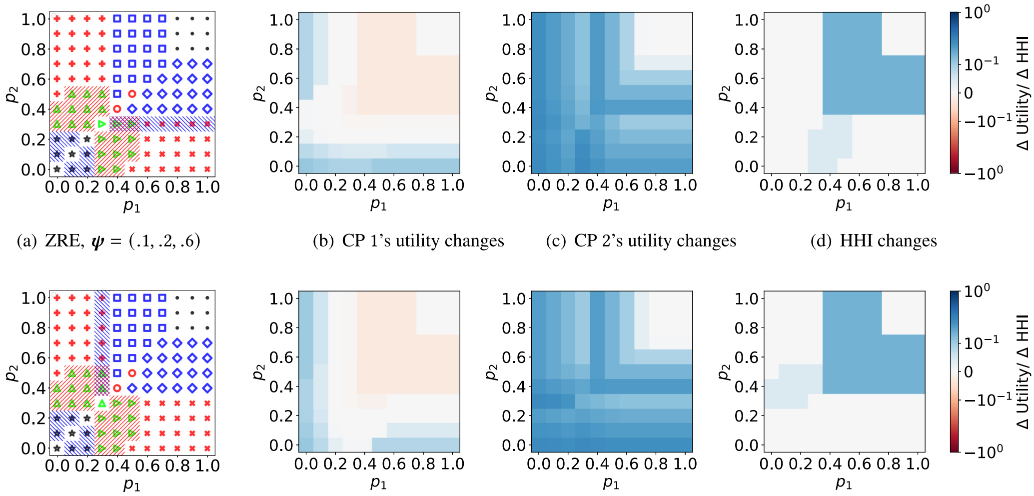
Fig. 6. ZRE under complementary duopoly (a, e). The differences in CPs' utilities and HHI when zero-rating is available and the market reaches equilibria, minus when zero-rating is not available (b, c, d, f, g, h). We have $\alpha = 0.5$, $c = 0.5$, $\boldsymbol{\psi} = (0.2, 0.4, 0.4)$, $\boldsymbol{\delta} = (1.0, 1.0)$, and $\boldsymbol{q} = (0.4, 1.0)$.

the high-value CP is more willing to zero-rate as it wants to increase its small initial market share, and it can afford that for most ISP prices, since it has a high value.

Figure 6 also shows the changes of CPs' utilities and the HHI corresponding to Figures 6(a) and 6(e) when $\boldsymbol{\phi}$ is being varied. When low-value CP 1 is the one with the smaller market share, ZRE causes its utility to slightly drop when CP 1 cannot afford to zero-rate while CP 2 can. Since this decrease is small, it may not be observable in Figure 6(b). However, when low-value CP 1 has a higher market share than CP 2, it also will have more elastic customers. When only CP 2 zero-rates, it will attract those customers from CP 1 and as a result, CP 1 will have a major utility drop (the red regions in Figure 6(f)). The utility of high-value CP 2 is generally increased compared to the case where no zero-rating is allowed. Note that CP 1's utility is higher when its baseline market share is higher, but Figure 6 merely illustrates the difference in utilities when zero-rating is available minus when it is not.

The only scenario where zero-rating can potentially have some benefit in terms of the HHI is shown in Figure 6(h), as the introduction of zero-rating increases the market share of the CP with lower baseline market share. Practically, this scenario translates to a case where a CP that is making more profit per customer has a smaller initial market, and it can directly increase its market share by reducing its profitability to move closer to the other CP. However, it is not clear that zero-rating is needed to address the market share imbalance and hence long term consumer benefit, since it introduces a utility loss to the low-value CP.

Figure 7 shows the impact of $\boldsymbol{\psi}$ on ZRE, CPs' utilities, and HHI. From (a, e), we observe that CPs are generally more willing to zero-rate the ISP with a higher baseline market share to attract more customers, and hence asymmetries are introduced to the graphs when $\psi_1 \neq \psi_2$. Therefore, compared to Figure 2, zero-rating pressure happens in a higher price range of the ISP with smaller ψ, since CP 1 is less willing to zero-rate with it without CP 2 zero-rating. The highest utility drop for CP 1 mainly happens when CP 2 establishes more zero-ratings than CP 1 while CP 2's utility generally increases. Furthermore, HHI in both cases is generally increasing.



(a) ZRE, $\boldsymbol{\psi} = (.1, .2, .6)$ (b) CP 1's utility changes (c) CP 2's utility changes (d) HHI changes

(e) ZRE, $\boldsymbol{\psi} = (.1, .6, .2)$ (f) CP 1's utility changes (g) CP 2's utility changes (h) HHI changes

Fig. 7. ZRE under complementary duopoly (a, e). The differences in CPs' utilities and HHI when zero-rating is available and the market reaches equilibria, minus when zero-rating is not available (b, c, d, f, g, h). We have $\alpha = 0.5$, $c = 0.5$, $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, $\boldsymbol{\delta} = (1.0, 1.0)$, and $\boldsymbol{q} = (0.4, 1.0)$.

Takeaways: As the baseline market share of low-value (high-value) CP increases (decreases), zero-rating mostly harms low-value CP's utility compared to the case where it is not available. With zero-rating available, high-value CP affords and has more incentives to zero-rate, and may attract elastic customer of low-value CP. As the baseline market shares of two ISPs become asymmetrical, CPs are more willing to zero-rate with the ISP with higher market share, where high-value CP affords to do so more than its low-value competitor, resulting in utility gains for high-value CP at the expense of utility loss for the low-value one.

Table 2 summarizes how different parameters impact CPs' average utilities and market shares over different ISP prices when zero-rating is available versus when it is not. This average is computed for all the graphs in this section, were the utility and market share values for 121 points were recorded over $p_1, p_2 \in \{0.0, 0.1, 0.2, \ldots, 1.0\}$. We see that regardless of parameter choice, aside from some exceptions, zero-rating mostly favors the utility and market share of high-value CP 2, while it harms those of low-value CP 1. Moreover, even though CP 1's average utility sometimes increases, its utility drops in a large ISP price range as well, and its gain is not as major as CP 2's average utility gain. As we saw in this section, the utility gains for CP 1 mostly occur in low ISP prices where both CPs afford to zero-rate. However, in higher ISP prices where the competition increases and CP 2 is mostly the only CP who zero-rates, there is a big range where CP 1's utility drops.

6 RELATED WORK

Cheng et al. [34] and Ma [35] consider the case where CPs bargain with the monopolistic ISP to obtain exclusive priority for their traffic; CPs are charged a fee only if they opt for priority, and users can access one content provider exclusively. Ma [35] studies a market of multiple CPs and ISPs, and assumes ISPs are always willing to offer exclusive priorities, while CPs are the decision-makers. While they both define a fixed market share for CPs, Cheng et al. incorporate consumer surplus for a case of monopolistic ISP and find that premium peering leaves content providers worse off, but Ma assumes ISPs are always willing to offer exclusive priorities, while CPs are the

Table 2. Impact of Parameters on CPs' Average Utilities After ZER Compared to When Zero-rating Is Not Available

Parameter changes against benchmark		Low-value CP's avg. of utility	Low-value CP's avg. of market share	High-value CP's avg. of utility	High-value CP's avg. of market share
benchmark parameters	—	↑	↓	↑	↑
bandwidth usage coefficient	decrease ($c = 0.2$)	↑	↓	↑	↑
	increase ($c = 0.8$)	↓	↓	↑	↑
user elasticity	decrease ($\alpha = 0.2$)	↑	↓	↑	↑
	increase ($\alpha = 0.8$)	↑	↓	↑	↑
CPs' baseline market shares	decrease low-value CP's baseline ($\phi = (0.1, 0.2, 0.6, 0.1)$)	↑	↓	↑	↑
	increase low-value CP's baseline ($\phi = (0.1, 0.6, 0.2, 0.1)$)	↑	↓	↑	↑
ISPs' baseline market shares	decrease ISP 1's baseline ($\psi = (0.2, 0.2, 0.6)$)	↑	↓	↑	↑
	increase ISP 1's baseline ($\psi = (0.2, 0.6, 0.2)$)	↑	↓	↑	↑

The average is taken over different ISP prices $p_1, p_2 \in \{0, 0.1, \ldots, 1.0\}$. Benchmark parameters are $\alpha = c = 0.5$, $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, $\boldsymbol{\psi} = (0.2, 0.4, 0.4)$, $\boldsymbol{\delta} = (1.0, 1.0)$, and $\boldsymbol{q} = (0.4, 1.0)$.

decision-makers. In our work, we consider zero-rating decisions in the market of multiple CPs and ISPs and study the case where customers do not necessarily use exclusive CPs. We consider both CPs' and ISPs' zero-rating decisions and show that zero-rating may cause market distortion by increasing the Herfindahl index in the market of CPs and usually leaves the low-value CP (startups with low incomes) worse off.

Choi and Kim [3] analyze the effects of net neutrality regulations on investment incentives for ISPs and CPs, and their implications for social welfare. Reggiani et al. [36] also model an Internet broadband provider that can offer a priority to two different content providers, low-value and high-value, and show that net neutrality regulations effectively protect innovation done at the edge by small content providers. Zhang et al. [37] builds a two-class service model to analyze the consumers' traffic demand under the sponsored data plan with consideration of QoS, and characterize the interaction between CPs and a monopolistic ISP. Shirmali [38] considers surplus extraction by a monopolistic ISP who controls the medium of information transfer between application developers and consumers, and shows that net neutrality is necessary to ensure maximal benefit to the society. Wong et al. [39] formulate an analytical model of the user, CP, and ISP interactions and derives their optimal behaviors. They show that zero-rating disproportionately benefits less cost-sensitive CPs and more cost-sensitive users, exacerbating disparities among CPs but reducing disparities among users. While the aforementioned models consider a market of monopolistic ISP and duopolistic CPs in which users access exclusively one content provider, our model extends to a larger market of ISPs and CPs where users are not required to access content providers exclusively.

Some previous studies focus on abolishing net neutrality under zero-rating. For instance, the authors in References [40–42] analyze zero-rating incentives of a monopolistic ISP in a homogeneous market of customers, and how different zero-rating equilibria impacts social welfare. Jullien et al. [43] discuss the elasticity of users and mainly focus on the case of a monopoly network with inelastic participation of consumers. Phalak et al. [44] also study a market of one ISP and two CPs under zero-rating, where the ISP determines the prices and the CPs make the zero-rating decisions. While these works all focus on a monopolistic ISP market, we extend our study to larger markets where ISPs and CPs are both decision-makers. In our model, customers can have as many CPs as they desire, and our main focus is on how zero-rating impacts innovations in the market and we do not analyze the customers' side in detail.

Some other works study real-world markets that have established zero-rating. Mathur et al. [45] analyze network usage data in South Africa and show that where usage-based billing is prevalent and data costs are high, users are cost-conscious where 90% of users consumed twice as much data when they do not pay for ISP bandwidth compared to when they have a usage-based mobile connection. Chen et al. [46] also collect a dataset and by analyzing zero-rating WhatsApp on Cell-C's network and zero-rating twitter on MTN's network, they find that zero-rating increases overall usage of the WhatsApp on Cell-C and Twitter on MTN network while it decreases it on most other providers. While in our work we use synthetic parameters to test our model, our final results and takeaways are not qualitatively impacted by parameter choice, albeit our model is flexible to use real-world parameters as a future direction.

7 DISCUSSIONS AND CONCLUSIONS

This article explores a controversial and unsettled aspect of net neutrality by analyzing zero-rating decisions in a market of multiple CPs and ISPs, and their impact on growing businesses and incumbents. We model the zero-rating decisions available between CPs and ISPs and find the *zero-rating equilibria* resulted in the market. By mainly focusing on CP's side of the market and analyzing the Herfindahl index, we have theoretically shown the distortions in the market may increase when zero-rating is available. We further numerically and qualitatively analyze the impact of zero-rating on CPs with different values and show that zero-rating typically disadvantages low-value CPs and could stunt the innovations. Our results strongly suggest that zero-rating can be harmful to competitiveness in a market, especially when the players have asymmetric market power and hence it should be disallowed under net neutrality.

Since there are different factors that impact ISPs' revenue, such as a price discount they might offer to CPs in case of zero-rating, we have considered a bargain between ISPs and CPs on price discount in Appendix B of this work and found the results are qualitatively the same. We consider the price of ISPs and the value of CPs as the units of their utilities, and by assuming that customers are probabilistically homogeneous, we analyze the market at both a macroscopic and a microscopic level. Due to the confidentiality of real-world data, we have made an exhaustive analysis of the market parameters, where our conclusions are general and can be applied to real-world scenarios. A future direction would be considering the customer's side of the market and analyzing the decisions they make to maximize their consumer surplus.

APPENDICES

APPENDIX A

Proof of Lemma 2.1. The additivity property holds because the users choose an ISP-CP pair based on the linear proportional form of Assumption 2, which induces the market shares of the providers in terms of linear functions in Equation (2). When zero-rating profiles, i.e., $\boldsymbol{\theta_i}$ or $\boldsymbol{\vartheta_j}$ are similar, the closed-form market share expressions keep the same form in terms of $\rho_{ij}(\Theta)$, while the baseline market shares of the providers, i.e., ϕ_i or ψ_j, are aggregated. □

Proof of Lemma 3.3. If the content provider with the lower value zero-rates, then zero-rating must be a strategy to cause its utility to increase. In that case, based on Equation (4), zero-rating would increase the utility of the high-value CP as well, since every characteristic of them are similar, only the high-value one has a higher q. In other words, if the low-value CP can afford zero-rating, so does the high-value CP. However, the opposite is not always true; if the high-value CP can afford zero-rating, the low-value CP would not necessarily afford it. Therefore, in the market of two CPs, possible zero-rating equilibria are either only the high-value CP zero-rate, or both zero-rate, or none zero-rates. □

Proof of Lemma 4.1. Suppose X_i is the fraction of users (market share) for CP i, and is equivalent to $\sum_{j\in(M)} \rho_{ij}$ when the total market size X is normalized to 1. We have

$$HHI = \sum_{i\in\mathbb{N}} X_i^2$$

and

$$\sigma^2 = \frac{\sum_{i\in\mathbb{N}} X_i^2}{|\mathbb{N}|} - \mu^2,$$

where the mean $\mu = \frac{\sum_{i\in\mathbb{N}} X_i}{|\mathbb{N}|} = \frac{1}{|\mathbb{N}|}$.

Therefore, we have

$$HHI = \frac{1}{|\mathbb{N}|} + \sigma^2|\mathbb{N}|.$$

Hence, the Herfindahl index has the minimum value of $\frac{1}{|\mathbb{N}|}$ when the variance of market shares is zero, and it increases as the variance increases. In a market of two content providers, this index grows when the gap between their market shares increases. □

Proof of Lemma 4.2. Without loss of generality, suppose we have two actual content providers in the system. Therefore, we will have 2^2 actual, dummy CP and CP bundles in our computations. Let us call the actual content providers CP 1 and CP 2, the dummy content provider CP 0, and the combination of CP 1 and CP 2 is called CP 3. We normalize the market size X to 1. When no zero-rating exists in the system, based on Equation (2), we have

$$X_{1j} = \phi_1\psi_j,\ X_{2j} = \phi_2\psi_j,\ X_{3j} = \phi_3\psi_j.$$

Therefore, given Equation (3):

$$X_1 = \sum_j X_{1j} + X_{3j} = \sum_j \psi_j(\phi_1 + \phi_3) = \phi_1 + \phi_3$$

and

$$X_2 = \sum_j X_{2j} + X_{3j} = \sum_j \psi_j(\phi_2 + \phi_3) = \phi_2 + \phi_3.$$

Hence, the Herfindahl index of the actual CPs would be

$$HHI = \frac{X_1^2 + X_2^2}{(X_1 + X_2)^2} = \frac{(\phi_1 + \phi_3)^2 + (\phi_2 + \phi_3)^2}{(\phi_1 + \phi_2 + 2\phi_3)^2}$$

$$\frac{\phi_i\psi_j\theta_{ij}}{\sum_{i'}\sum_{j'}\phi_{i'}\psi_{j'}\theta_{i'j'}}\alpha + \phi_i\psi_j(1-\alpha).$$

However, if everyone zero-rates in the system, then we have

$$X_{1j} = \phi_1\psi_j(1-\alpha) + \frac{\phi_1\psi_j}{\sum_{i'}\sum_{j'}\phi_{i'}\psi_{j'}\theta_{i'j'}}\alpha = \phi_1\psi_j(1-\alpha).$$

Let us define

$$A = 1 - \alpha + \frac{\alpha}{\sum_{i'}\sum_{j'}\phi_{i'}\psi_{j'}\theta_{i'j'}},$$

then we have

$$X_{1j} = A \times \phi_1\psi_j.$$

Similarly, for X_{2j} and X_{3j}, we have

$$X_{2j} = A \times \phi_2\psi_j,\ X_{3j} = A \times \phi_3\psi_j.$$

Therefore,

$$X_1 = \sum_j X_{1j} + X_{3j} = \sum_j \psi_j (A\phi_1 + A\phi_3) = A(\phi_1 + \phi_3)$$

and

$$X_2 = \sum_j X_{2j} + X_{3j} = \sum_j \psi_j (A\phi_2 + A\phi_3) = A(\phi_2 + \phi_3).$$

Hence, the Herfindahl index of the actual CPs would be

$$HHI = \frac{X_1^2 + X_2^2}{(X_1 + X_2)^2}$$

$$= \frac{A^2(\phi_1 + \phi_3)^2 + A^2(\phi_2 + \phi_3)^2}{A^2(\phi_1 + \phi_2 + 2\phi_3)^2} = \frac{(\phi_1 + \phi_3)^2 + (\phi_2 + \phi_3)^2}{(\phi_1 + \phi_2 + 2\phi_3)^2}.$$

Which is the same Herfindahl index of the case where no zero-rating is allowed in the system. □

Proof of Theorem 4.3. Based on Lemma 3.3, for any two CPs with different values, the possible zero-rating strategies in the equilibria are either the high-value content provider zero-rates, or both of them zero-rate, or neither of them zero-rate. In the last two cases, the Herfindahl index would be the same according to Lemma 4.2. Further, when no one zero-rates, the market shares would be the same as when no zero-rating is allowed in the system, therefore the Herfindahl index would be the same. In the first case, based on Corollary 2.2, the high-value content provider would attract more customers if it zero-rates. Since the high-value content provider is the one with the higher baseline market share, after it zero-rates, its market share would further increase from its baseline. Therefore, the gap between market shares increases as well. This could extend to multiple CPs where the variance of the market shares would increase in this case. Hence, based on Lemma 4.1, the Herfindahl index increases as well. □

Proof of Theorem 4.4. When a low-value CP cannot zero-rate, any zero-rating relation that its high-value opponent establishes will decrease its utility. The reason is that based on Equation (2), in case there is any zero-rating relations in the market ($\Theta \neq \mathbf{0}$), the market share of a pair of CP i and ISP j is computed from

$$\rho_{ij}(\Theta) = \frac{\phi_i \psi_j \theta_{ij}}{\sum_{i'} \sum_{j'} \phi_{i'} \psi_{j'} \theta_{i'j'}} \alpha + \phi_i \psi_j (1 - \alpha).$$

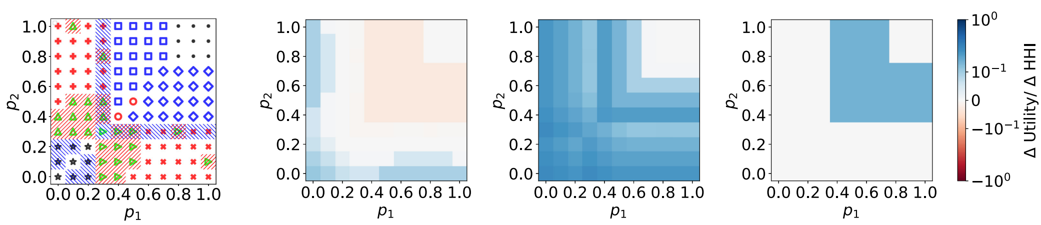
If low-value CP i does not zero-rate, then $\theta_{ij} = 0 \forall j \in \mathbb{M}$. Therefore, the first term in Equation (2) will be zero and as a result, if $\theta_{i'j} = 1 \, \exists j \in \mathbb{M}$, the elastic users of CP i will move to CP i'. Based on Equation (4), the utility of CP i produced from each ISP j will be computed from the term $U_i^j(\Theta) = q_1 \mathbb{X}_{ij}(\Theta).c$, and as $\mathbb{X}_{ij} = X\rho_{ij}$ decreases, CP i's utility decreases as well.

However, if CP i' zero-rates with ISP j, then the same analysis cannot be applied to its utility. Even though based on Corollary 2.2, in case $\theta_{i'j} = 1 \, \exists j \in \mathbb{M}$ the market share of CP i' increases, as it is also paying the bandwidth price of p_j to ISP j, its utility computation is different than when it is not zero-rating ($U_{i'}^j(\Theta) = (q_{i'} - \delta_j p_j)\mathbb{X}_{i'j}(\Theta)$ if $\theta_{i'j} = 1$ versus $U_{i'}^j(\Theta) = (q_{i'})\mathbb{X}_{i'j}(\Theta).c$ if $\theta_{i'j} = 0$). However, based on the definition of ZRE, we can prove its utility would not decrease. In ZRE, if low-value CP does not zero-rate while the high-value CP does, i.e., $\theta_{ij} = 0$ and $\theta_{i'j} = 1$, then it would not be a ZRE if it decreases high-value CP's utility compared to $\theta_{ij} = 0$ and $\theta_{i'j} = 0$. Therefore, when low-value CP i does not zero-rate, any zero-rating relation of CP i' after ZRE will either increase its utility or keep it unchanged. □

APPENDIX B

In this Appendix, we consider the case where ISPs decide what price discounts ($\boldsymbol{\delta}$) to offer for zero-rating. These discounts are in the spirit of bulk discounts, as the CP would be paying for *all* of its users in bulk. For simplicity of computations, we assume each ISP j chooses δ_j from the set $\{0.0, 0.1, \ldots, 1.0\}$. We define $\bar{\boldsymbol{\delta}}_j \triangleq \{0.0, 0.1, \ldots, 1.0\} - \{\delta_j\}$, and we denote the discount strategy of all ISPs except for ISP j by $\boldsymbol{\delta}_{-j}$. A discount profile $\boldsymbol{\delta}$ is defined as a *Nash equilibrium* if there exists a ZRE built on that, and for the resulting ISP revenues we have $R_j|_{(\delta_j, \boldsymbol{\delta}_{-j})} \geq R_j|_{(\bar{\boldsymbol{\delta}}_j, \boldsymbol{\delta}_{-j})}$ for all ISP j. Since there might be multiple values of $\boldsymbol{\delta}$ satisfying this condition, we make an arbitrary assumption choose $\boldsymbol{\delta}$ to be the maximum 2-norm among those who lead to equilibria. We further assume that ISP 1 is the tie breaker. More specifically, if we have a tie between different vectors of $\boldsymbol{\delta}$, then the one will be chosen that has a higher δ_1.

We again use our benchmark scenario on Figure 2, where we had $\boldsymbol{q} = (0.4, 1.0)$, $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, $\boldsymbol{\psi} = (0.1, 0.45, 0.45)$, $\alpha = 0.5$, and $c = 0.5$. Figure 8(a) depicts the final equilibria in the market where both CPs and ISPs make zero-rating decisions, and Figure 8 depicts CPs' utilities and HHI changes corresponding to Figure 8(a). Furthermore, Table 3 shows the discount profiles $\boldsymbol{\delta}$ of the ISPs. Compared to Figure 2 where $\boldsymbol{\delta} = \mathbf{1}$ and it is remained constant, we observe that the number of zero-rating relations increases after ZRE. The reason is that as Table 3 shows, the ISPs may offer discounts for some price profiles so that they attract CPs to zero-rate with them, and as a result expand their market. Note that these price discounts are derived from Nash equilibria and as ISPs are also decision maker agents, they choose their discounts high enough to maximize their rewards given a particular profile the rest of the market has.



(a) ZRE, varying $\boldsymbol{\delta}$ (b) CP 1's utility changes (c) CP 2's utility changes (d) HHI changes

Fig. 8. ZRE (a) and the differences in CPs' utilities and HHI when zero-rating is available and the market reaches equilibria, minus when zero-rating is not available (b, c, d). We have $\alpha = 0.5$, $c = 0.5$, $\boldsymbol{\phi} = (0.1, 0.4, 0.4, 0.1)$, $\boldsymbol{\psi} = (0.2, 0.4, 0.4)$, and $\boldsymbol{q} = (0.4, 1.0)$.

Table 3. The Discount Profile $\boldsymbol{\delta}$ of ISPs Under Complementary Duopoly ZRE of Figure 8

p_2 \ p_1	0.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 0.9)	(1.0, 0.8)	(1.0, 0.7)	(1.0, 0.6)	(1.0, 0.5)
0.1	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 0.9)	(1.0, 0.8)	(1.0, 0.7)	(1.0, 0.6)	(1.0, 0.5)
0.2	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 0.8)	(1.0, 0.7)	(1.0, 0.6)	(1.0, 1.0)
0.3	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 0.7)	(1.0, 1.0)	(1.0, 0.6)
0.4	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 0.6)
0.5	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 0.9)	(1.0, 0.8)	(1.0, 0.7)	(1.0, 0.6)
0.6	(0.9, 1.0)	(0.9, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 0.8)	(1.0, 0.7)	(1.0, 1.0)
0.7	(0.8, 1.0)	(0.8, 1.0)	(0.8, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(0.9, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 1.0)
0.8	(0.7, 1.0)	(0.7, 1.0)	(0.7, 1.0)	(0.7, 1.0)	(1.0, 1.0)	(0.8, 1.0)	(0.8, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(1.0, 0.9)	(1.0, 0.8)
0.9	(0.6, 1.0)	(0.6, 1.0)	(0.6, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(0.7, 1.0)	(0.7, 1.0)	(1.0, 1.0)	(0.9, 1.0)	(0.9, 0.9)	(0.9, 0.8)
1.0	(0.5, 1.0)	(0.5, 1.0)	(1.0, 1.0)	(0.6, 1.0)	(0.6, 1.0)	(0.6, 1.0)	(1.0, 1.0)	(1.0, 1.0)	(0.8, 1.0)	(0.8, 0.9)	(0.8, 0.8)

As in Figure 8, we observe the difference in utilities of CPs, and the HHI after equilibria when zero-rating is available minus when it is not. We notice that the general pattern of the utility loss for low-value CP 1 and the utility gain for high-value CP 2 are similar to the benchmark in Figure 3, with slight changes due to ISP price discounts. The reason for these changes is also that offering a discount would have a similar impact on CPs' side of the market as if the ISP has a cheaper price.

We conclude that even when ISPs make decisions on what price discounts to offer, the general behavior of the market and the notion that zero-rating harming the low-value CP and favoring the high-value one remains unchanged; hence, we have included this analysis in the Appendix for completeness. Note that since the graphs for different parameter selections are also qualitatively similar to those in Section 5 and mostly follow the same trends, we have not included them in the Appendix.

REFERENCES

[1] Angele A. Gilroy. 2011. *Access to Broadband Networks: The Net Neutrality Debate.* DIANE Publishing.

[2] Tim Wu. 2003. Network neutrality, broadband discrimination. *J. Telecomm. High Tech. L.* 2 (2003), 141.

[3] Jay Pil Choi and Byung-Cheol Kim. 2010. Net neutrality and investment incentives. *RAND J. Econ.* 41, 3 (2010), 446–471.

[4] Joshua S. Gans. 2015. Weak versus strong net neutrality. *J. Regul. Econ.* 47, 2 (2015), 183–200.

[5] Marc Bourreau, Frago Kourandi, and Tommaso Valletti. 2015. Net neutrality with competing internet platforms. *J. Industr. Econ.* 63, 1 (2015), 30–73.

[6] Iain Marlow. U.S. regulator adopts Internet traffic rules. Retrieved from https://www.theglobeandmail.com/technology/us-regulator-adopts-internet-traffic-rules/article1320850/.

[7] Open Internet Order. 2015. Supra note 13, at 5666–69 paras.

[8] FCC's Restoring Internet Freedom Order. Retrieved from https://www.fcc.gov/document/restoring-internet-freedom-order-takes-effect.

[9] Rishi Iyengar. India now has the "world's strongest" net neutrality rules. Retrieved from https://money.cnn.com/2018/07/12/technology/india-net-neutrality-rules-telecom/index.html.

[10] Internet Freedom Foundation. 2018. Net neutrality. Retrieved from https://internetfreedom.in/issues-neutrality/.

[11] Arturo J. Carrillo. 2015. Having your cake and eating it too: Zero-rating, net neutrality, and international law. *Stan. Tech. L. Rev.* 19 (2015), 364.

[12] Chris Ziegler. T-Mobile's "Music Freedom" is a great feature—and a huge problem. Retrieved from https://www.theverge.com/2014/6/18/5822996/t-mobile-music-freedom-net-neutrality.

[13] Dan Seifert. T-Mobile's latest "Un-carrier" feature: Rhapsody Unradio, an odd streaming music service. Retrieved from https://www.theverge.com/2014/6/18/5819804/t-mobile-unradio-rhapsody-free-internet-radio-service.

[14] Greg Kumparak. T-Mobile stops counting netflix, HBO, hulu, and other video streams against your data usage. Retrieved from https://techcrunch.com/2015/11/10/t-mobile-stops-counting-netflix-hbo-hulu-and-other-video-streams-against-your-data-usage/.

[15] Andrew Tonner. Verizon joins AT&T in this controversial net neutrality practice. Retrieved from https://www.fool.com/investing/general/2016/01/21/verizon-joins-att-in-this-controversial-net-neutra.aspx.

[16] Darrell M. West. 2015. Digital Divide: Improving Internet Access in the Developing World Through Affordable Services and Diverse Content. Retrieved from https://www.brookings.edu/wp-content/uploads/2016/06/West_Internet-Access.pdf.

[17] Luca Belli. 2017. Net neutrality, zero rating and the minitelisation of the internet. *J. Cyber Policy* 2, 1 (2017), 96–122.

[18] Niloofar Bayat, Richard Ma, Vishal Misra, and Dan Rubenstein. 2021. Zero-rating and net neutrality: Who wins, who loses? *ACM SIGMETRICS Perform. Eval. Rev.* 48, 3 (2021), 130–135.

[19] Stephen A. Rhoades. 1993. The Herfindahl-Hirschman index. *Fed. Res. Bull.* 79 (1993), 188.

[20] The United States Department of Justice. [n.d.]. Herfindahl-Hirschman Index. Retrieved from https://www.justice.gov/atr/herfindahl-hirschman-index.

[21] Daniel McFadden. 1980. Econometric models for probabilistic choice among products. *J. Business* 53, 3 (July 1980).

[22] R. Duncan Luce. 1959. *Individual Choice Behavior: A Theoretical Analysis.* Wiley, New York, NY.

[23] L. Anania and R. J. Solomon. 1997. *Flat—The Minimalisit Price, in Internet Economics.* L. W. McKnight and J. P. Bailey (eds.). MIT Press Cambridge, MA, 91–118.

[24] L. Segall. 2011. Verizon ends unlimited data plan. *CNN Money* (July 6, 2011).

[25] P. Taylor. 2011. AT&T imposes usage caps on fixed-line broadband. *Financial Times* (Mar. 14, 2011).

[26] John Nash. 1951. Non-cooperative games. *Ann. Math.* (1951), 286–295.

[27] Martin Nowak and Karl Sigmund. 1993. A strategy of win-stay, lose-shift that outperforms tit-for-tat in the Prisoner's Dilemma game. *Nature* 364, 6432 (1993), 56.
[28] Keith Cowling and Dennis C. Mueller. 1978. The social costs of monopoly power. *Econ. J.* 88, 352 (1978), 727–748.
[29] USDOJ. Retrieved from https://www.justice.gov/atr/herfindahl-hirschman-index.
[30] William M. Landes and Richard A. Posner. 1997. Market power in antitrust cases. *J. Reprints Antitrust L. Econ.* 27 (1997), 493.
[31] Horizontal Merger Guidelines. 2010. Retrieved from https://www.justice.gov/atr/horizontal-merger-guidelines-08192010.
[32] Itzhak Gilboa and Eitan Zemel. 1989. Nash and correlated equilibria: Some complexity considerations. *Games Econ. Behav.* 1, 1 (1989), 80–93.
[33] Vincent Conitzer and Tuomas Sandholm. 2008. New complexity results about Nash equilibria. *Games Econ. Behav.* 63, 2 (2008), 621–641.
[34] Hsing Kenneth Cheng, Subhajyoti Bandyopadhyay, and Hong Guo. 2011. The debate on net neutrality: A policy perspective. *Info. Syst. Res.* 22, 1 (2011), 60–82.
[35] Richard T. B. Ma. 2017. Pay or perish: The economics of premium peering. *IEEE J. Select. Areas Commun.* 35(2) (Feb. 2017), 353 – 366.
[36] Carlo Reggiani and Tommaso Valletti. 2016. Net neutrality and innovation at the core and at the edge. *Int. J. Industr. Org.* 45 (2016), 16–27.
[37] Liang Zhang, Weijie Wu, and Dan Wang. 2015. Sponsored data plan: A two-class service model in wireless data networks. In *ACM SIGMETRICS Performance Evaluation Review*, Vol. 43. ACM, 85–96.
[38] Gireesh Shrimali. 2008. Surplus extraction by network providers: Implications for net neutrality and innovation. *Telecommun. Policy* 32, 8 (2008), 545–558.
[39] Carlee Joe-Wong, Soumya Sen, and Sangtae Ha. 2018. Sponsoring mobile data: Analyzing the impact on internet stakeholders. *IEEE/ACM Trans. Netw.* 26, 3 (2018), 1179–1192.
[40] Robert Somogyi et al. 2017. The economics of zero-rating and net neutrality. Unpublished Manuscript. Université Catholique de Louvain, Center for Operations Research and Econometrics (CORE).
[41] Thomas Jeitschko, Soo Jin Kim, and Aleksandr Yankelevich. 2017. Zero-rating and vertical content foreclosure. Unpublished Manuscript. Michigan State University, East Lansing.
[42] Martin Kies. 2017. Impacts of sponsored data on infrastructure investments and welfare.
[43] Bruno Jullien and Wilfried Sand-Zantman. 2018. Internet regulation, two-sided pricing, and sponsored data. *Int. J. Industr. Org.* 58 (2018), 31–62.
[44] Kunal Phalak, D. Manjunath, and Jayakrishnan Nair. 2019. Zero rating: The power in the middle. *IEEE/ACM Trans. Netw.* 27, 2 (2019), 862–874.
[45] Arunesh Mathur, Brent Schlotfeldt, and Marshini Chetty. 2015. A mixed-methods study of mobile users' data usage practices in South Africa. In *Proceedings of the ACM International Joint Conference on Pervasive and Ubiquitous Computing*. ACM, 1209–1220.
[46] Ava Chen, Nick Feamster, and Enrico Calandro. 2017. Exploring the walled garden theory: An empirical framework to assess pricing effects on mobile data usage. *Telecommun. Policy* 41, 7–8 (2017), 587–599.

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