Interconnecting Eyeballs to Content: A Shapley Value Perspective on ISP Peering and Settlement

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Abstract

Internet service providers (ISPs) must interconnect to provide global Internet connectivity to users. The payment structure of these interconnections are often negotiated and maintained via bilateral agreements. Current differences of opinion in the appropriate revenue model in the Internet has on occasion caused ISPs to de-peer from one another, hindering network connectivity and availability.

Our previous work demonstrates that the Shapley value has several desirable properties, and that if applied as the revenue model, selfish ISPs would yield globally optimal routing and interconnecting decisions. In this paper, we focus our investigation of Shapley value in networks with two basic classes of ISP: content and eyeball. In particular, we analyze the revenue distribution between ISPs with elastic and inelastic customer demands, and calculate the bilateral payments between ISPs that implement the Shapley revenue. Our results illustrate how ISP revenues are influenced by different demand models. In particular, the marginal revenue lost by de-peering for an eyeball ISP with inelastic demand is inversely proportional to the square of its degree of connectivity to content ISPs. In practice, these results provide a guideline for ISPs, even in peering relationships, to negotiate bilateral payments and for regulatory institutions to design pricing regulations.

Index Terms

Shapley value, paid-peering, bilateral agreements.

I. Introduction

The Internet consists of thousands of interconnected ISPs, with each ISP interested in maximizing its own profits. Interconnection agreements, often negotiated bilaterally between ISPs, are much more complex and problematic today than they were several years back, when Tier 1 ISPs would willingly all peer with one another, and other interconnection agreements could easily be classified as *transit* or *peering*. Today, because of the heterogeneity in ISPs, simple peering agreements are not always satisfactory to all parties involved, and *paid peering* [1] has naturally emerged as the preferred form of settlement among the heterogeneous ISPs. The transition to paid peering has not always been a graceful one. For example, the ISP named Level 3 unilaterally terminated its "settlement free" peering relationship with Cogent on October 5, 2005. This disruption resulted in at least 15% of the Internet to be unreachable for the users who utilized either Level 3 or Cogent for Internet access. Although both companies restored peering connections several days later with a new on-going negotiation, Level 3's move against Cogent exhibited an escalation of the tensions that threaten settlement-free peering.

Faratin et al. [1] view today's Internet as containing two classes of ISPs: *content* and *eyeball*. Content ISPs specialize in providing hosting and network access for end-customers and commercial companies that offer content, such as Google, Yahoo, and YouTube. Eyeball ISPs, such as AT&T and Verizon, specialize in delivery to hundreds of thousands of end-customers, i.e., supporting the last-mile connectivity. Most *commerce generating*[1] traffic flows from the content ISPs to the eyeball ISPs. An open problem, which often is the centerpiece of the *network neutrality debate* [2], [3], [4], involves determining an appropriate compensation structure that distributes revenues "fairly" between these two classes of peering ISPs. Without solving this problem, we can expect future peering disputes that will lead to further de-peering disruptions within the Internet.

In our previous work [5], we explored the application of *Shapley value* [6], [7], a well-known economic concept originated from coalition games [8], [9], [10], to a general ISP setting. We showed that if profits were shared as perscribed by the Shapley value mechanism, not only would the set of desirable "fair" properties inherent to the Shapley solution exist, but also that the selfish behaviors of the ISPs would also yield globally optimal routing and interconnecting decisions.

This paper provides a preliminary exploration into the implications of sharing profits as perscribed by our application of Shapley Value in [5] to networks whose ISP structure fits the content/eyeball classification of [1]. To focus specifically on the content/eyeball dimension of the problem, we model ISP connectivity as a bipartite graph with content ISPs on one side and eyeball ISPs on the other. Using this bipartite structure, we consider separately the cases where customers who are *inelastic* and are assigned to a single eyeball ISP, and customers who are *elastic* and can choose the eyeball ISP from a set whose size is larger than one, from which they receive connectivity. This simple yet elegant formulation omits some practical concerns like transit costs, but permits a low-complexity algorithm for computing the Shapley value (which, in general networks has a high

[1]Peer-to-Peer traffic, which flows between eyeball ISPs, is outside the scope of this study

complexity), allowing us to specifically focus on how the roles of the content and eyeball ISPs, and the various relationships they have with their respective customers impact their profitability.

Our results are:

- We obtain closed-form Shapley revenues for all ISPs and give a bilateral payment implementation in terms of the percentage of ISP customer revenues.
- We show that when customers are inelastic, the Shapley revenue is separable: each eyeball ISP's revenue is proportional to its customer size, and is independent of other eyeball ISPs' sizes. Each eyeball ISP controls $d/(d+1)$ fraction of the generated revenue, where d is the number of connected content ISPs.
- We quantify the marginal loss for an eyeball ISP with inelastic customer demand. We show that the percentage of revenue loss for an eyeball ISP is inversely proportional to the square of the number of content ISPs it currently connects to.
- We show that when customers are elastic, content ISPs and eyeball ISPs have the same role in revenue distribution. Under a complete partite topology, the revenue ratio of both groups of ISPs equals the inverse of the ratio of number of ISPs in each group.

We believe that the bilateral payment solution gives a guideline for paid peering agreements for ISPs to negotiate based on the characteristics of customer demand, content distribution and ISP topologies.

II. Shapley value and properties

Here, we briefly introduce the concept of Shapley value and its use under our ISP revenue distribution context. We follow the notations in [5]. We consider a network system comprised of a set of ISPs denoted as $\mathcal{N}$. $N = |\mathcal{N}|$ denotes the number of ISPs in the network. We call any nonempty subset $\mathcal{S} \subseteq \mathcal{N}$ a *coalition* of the ISPs. Each coalition can be thought of as a sub-network that might be able to provide partial services to their customers. The network system is defined as $(\mathcal{N}, v, E)$. E denotes the set of directed links between the ISPs. The graph $G = (\mathcal{N}, E)$ defines the ISP topology of the network. We denote $G_{\mathcal{S}}$ as the subgraph of G induced by $\mathcal{S}$, defined by $G_{\mathcal{S}} = (\mathcal{S}, E_{\mathcal{S}})$, where $E_{\mathcal{S}} = \{(i,j) \in E : i,j \in \mathcal{S}\}$. $G_{\mathcal{S}}$ is the ISP topology formed by the coalition $\mathcal{S}$.

We denote v as the *worth function*, which measures the monetary payments produced by the sub-networks formed by all coalitions. In other words, for any coalition $\mathcal{S}$, $v(\mathcal{S})$ defines the revenue generated by the sub-network formed by the set of ISPs $\mathcal{S}$. In particular, v measures the aggregate end-payments each ISP in a coalition obtains in a specific topology as

$$v(\mathcal{S}, E_{\mathcal{S}}) = \sum_{i \in \mathcal{S}} P_i(\mathcal{S}, E_{\mathcal{S}}), \tag{1}$$

where $P_i(\mathcal{S}, E_{\mathcal{S}})$ is the end-payment collected by ISP i in a coalition topology $G_{\mathcal{S}} = (\mathcal{S}, E_{\mathcal{S}})$. To avoid the redundancy in the notation, we drop $E_{\mathcal{S}}$ and denote $v(\mathcal{S})$ as the worth function for any fixed topology. Through the worth function v, we can measure the contribution of an ISP to a group of ISPs as the following.

Definition *1:* The *marginal contribution* of ISP i to a coalition $\mathcal{S} \subseteq \mathcal{N} \backslash \{i\}$ is defined as $\Delta_i(v, \mathcal{S}) = v(\mathcal{S} \cup \{i\}) - v(\mathcal{S})$.

Proposed by Lloyd Shapley [6], [7], the Shapley value serves as an appropriate mechanism for ISPs to share revenues.

Definition *2:* The *Shapley value* φ is defined by

$$\varphi_i(\mathcal{N}, v) = \frac{1}{N!} \sum_{\pi \in \Pi} \Delta_i(v, S(\pi, i)) \qquad \forall\ i \in \mathcal{N}, \tag{2}$$

where Π is the set of all $N!$ orderings of $\mathcal{N}$ and $S(\pi, i)$ is the set of players preceding i in the ordering π.
The Shapley value of an ISP can be interpreted as the expected marginal contribution $\Delta_i(v, \mathcal{S})$ where $\mathcal{S}$ is the set of ISPs preceding i in a uniformly distributed random ordering. The Shapley value depends only on the values $\{v(\mathcal{S}) : \mathcal{S} \subseteq \mathcal{N}\}$. The Shapley value satisfies a bunch of desirable efficiency and fairness properties [5].

We showed in [5] that the Shapley value mechanism also induces global Nash equilibra that are globally optimal for routing and interconnecting. In our prior work, we assumed an oracle that performed global revenue (re)allocation based on the Shapley value. That assumption however has clear practical and regulatory limitations. In this paper, we focus on ISP interconnecting and revenue distribution amongst *peers*. We assume the routing costs are negligible compared to the revenue obtained from providing services. Nevertheless, our framework can always be extended to include an orthoganal direction of routing decisions and costs.

III. The ISP model

We follow the categorization of ISPs by Faratin et al. [1] as two basic types [1]: content ISPs and eyeball ISPs. The set of ISPs is defined as $\mathcal{N} = \mathcal{C} \cup \mathcal{B}$, where $\mathcal{C} = \{C_1, \cdots, C_{|\mathcal{C}|}\}$ denotes the set of content ISPs and $\mathcal{B} = \{B_1, \cdots, B_{|\mathcal{B}|}\}$ denotes the set of eyeball ISPs. We denote $\mathcal{Q}$ as the set of contents provided by the set of content ISP $\mathcal{C}$. Each content ISP C_i provides a subset of the contents $Q_i \subseteq \mathcal{Q}$. For each content $q \in \mathcal{Q}$, we define a popularity factor k_q for that content, which is used to quantify the relative amount of demand that end-users are going to download this content. If only one content is provided by all content ISPs, we denote the popularity factor simply as k. We assume a size of x of total end-customer population in the network. Each eyeball ISP B_j attracts a portion x_j of the total population size. We assume the conservation of the end-customer population size, i.e. $x = \sum_{j=1}^{|\mathcal{B}|} x_j$. We also define two monetary factors α and β associated with end-customers and content providers. α can be regarded as the average monthly Internet access fee per customer. Similarly, β can be regarded as the average compensation paid by content providers to content ISPs per traffic demand unit. We denote r_{ij} as the traffic rate originated from content ISP C_i to eyeball ISP B_j.

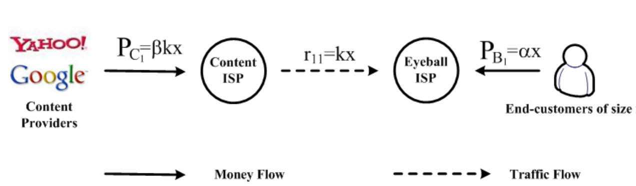

Fig. 1. A content ISP and an eyeball ISP.

Figure 1 illustrates the simplest scenario where one eyeball ISP owns the whole customer size x and one content ISP provides a single content, which attracts monthly download traffic rate of kx. We assume that the traffic rate is proportional to both the population size and the popularity factor of the content. The eyeball ISP's revenue equals αx (dollars); the content ISP's revenue equals $\beta k x$ (dollars).

A. Topology, Revenue and Traffic Model

We define the complete bipartite graph, which connects each content ISP with all other eyeball ISPs, as $\tilde{G} = (\mathcal{C} + \mathcal{B}, \tilde{E})$, where $\tilde{E} = \{(C_i, B_j) : C_i \in \mathcal{C}, B_j \in \mathcal{B}\}$. We are only interested in the set of links $E \subseteq \tilde{E}$ that connect content ISPs with eyeball ISPs. Links among eyeball ISPs and among content ISPs can be present in reality; however, due to the characteristics of the Internet traffic pattern, the traffic volume on these links are either relatively small or symmetric (peer-to-peer). Therefore, "charge free" peering agreements can be expected for building these links among the same type of ISPs, if necessary.

Under different topology and user demand models, the population size of each eyeball ISP differs. We define the *direct payment* received by an eyeball ISP B_j as a function of its realizing customer size x_j:

$$P_{B_j}(\mathcal{N}, E) = \alpha x_j, \tag{3}$$

where α is the monetary factor defined before.

We denote d_q as the number of content ISPs which provide content q, d_{B_j} as the number of content ISPs that connect to eyeball ISP B_j, and $d^q_{B_j}$ as the number of content ISPs that both connect to B_j and provide content q. In general, $d_{B_j} \neq 0$, because, otherwise, we can conceptually exclude B_j from the network system without affecting other ISPs. These three quantities are defined as follows.

$$d_q = \sum_{i=1}^{|\mathcal{C}|} \mathbf{1}_{\{q \in Q_i\}}, \qquad d_{B_j} = \sum_{i=1}^{|\mathcal{C}|} \mathbf{1}_{\{(C_i, B_j) \in E\}}, \tag{4}$$

$$d^q_{B_j} = \sum_{i=1}^{|\mathcal{C}|} \mathbf{1}_{\{q \in Q_i\}} \mathbf{1}_{\{(C_i, B_j) \in E\}}. \tag{5}$$

We assume the traffic rate r_{ij} (from C_i to B_j) as the following:

$$r_{ij} = \sum_{q \in Q_i} \frac{k_q}{d^q_{B_j}} x_j. \tag{6}$$

The above equation assumes that traffic rate is proportional to the customer size, the popularity of the contents and the inverse of number of content ISPs who are sharing the same content. This implies that eyeball ISPs download content from all available content ISPs uniformly. Finally, we define the *direct payment* received by each content ISP C_i as the following:

$$P_{C_i}(\mathcal{N}, E) = \beta \sum_{j:(i,j) \in E} r_{ij}, \tag{7}$$

where β is the average per traffic demand compensation paid by content providers.

Figure 2 illustrates a scenario with $|\mathcal{C}| = 3$, $|\mathcal{B}| = 2$, $E = \{(1,1), (2,2), (3,1), (3,2)\}$, and $\mathcal{Q} = \{1, 2\}$. The contents provided by three content ISPs are $Q_1 = \{1, 2\}, Q_2 = \{2\}$ and $Q_3 = \{1\}$ respectively. It also shows the traffic rates as well as the revenues of all ISPs.

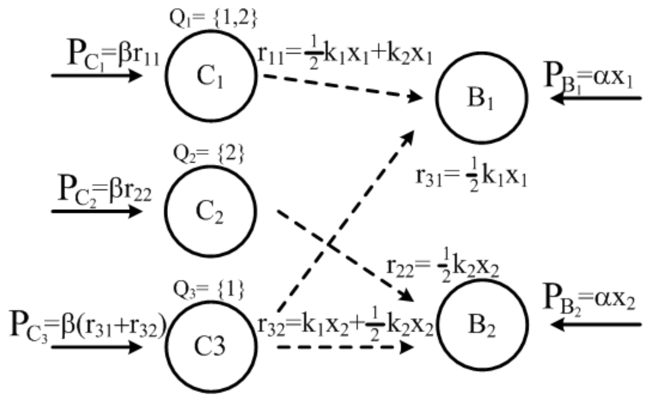

Fig. 2. An example of content and traffic model for the ISPs.

B. Customer Demand Model

To calculate the Shapley value for individual ISPs, we need to consider the contribution of each ISP to any coalition of a subset of the ISPs. Eyeball ISPs' immediate revenue come from end-customers; content ISPs' immediate revenue come from the traffic demand from their connected eyeball ISPs. Therefore, the customer demand pattern determines the revenues of the coalitions of the ISPs as well as the Shapley value for each ISP.

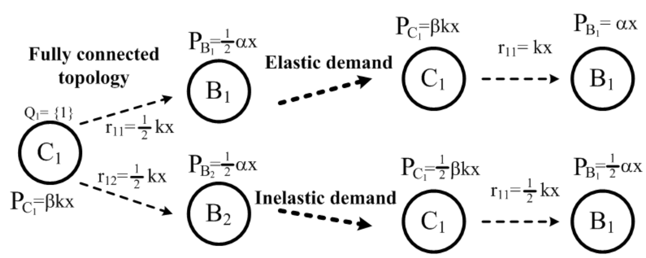

Fig. 3. Inelastic and elastic demand patterns.

Figure 3 illustrates a simple case with two eyeball ISPs and one content ISP, evenly sharing a customer size of x. We consider a coalition $\{C_1, B_1\}$ when B_2 de-peers with C_1. Under an *inelastic* demand model, the customer size of B_1 remains $x/2$, which implies that the customers of B_2 cannot move to B_1 due to physical constraints, e.g. geographic or regulatory constraints. In this case, the whole coalition loses half of the customers as well as half of the total revenue. Under an *elastic* demand model, the customers of B_2 shift to B_1 and the coalition's revenue remains the same when B_2 was connected. We can imagine that under the elastic demand case, the importance of eyeball ISPs is relatively small compared with that of the inelastic case, since the eyeball ISPs cannot control its end-customers. As a special case of a general result we will show in later sections, the Shapley value for the ISPs are $\varphi_{B_1} = \varphi_{B_2} = \frac{1}{2}\varphi_{C_1} = \frac{1}{4}(\alpha+\beta k)x$ and $\varphi_{B_1} = \varphi_{B_2} = \frac{1}{4}\varphi_{C_1} = \frac{1}{6}(\alpha+\beta k)x$ under the inelastic and elastic models respectively.

In the next sections, we model both inelastic and elastic customer demand in detail. Under both demand models, we quantify the Shapley values for each ISP and the bilateral transfers between ISPs which implement the Shapley value revenue distribution. We will explain the effects of demand elasticity, privileges of contents and topologies on Shapley value as well as the bilateral payments between ISPs.

IV. Inelastic Customer Demand

In this section, we consider an inelastic customer demand model. We assume that each eyeball ISP B_i attracts a fixed customer size of x_i. An eyeball ISP B_j obtains direct revenue αx_j from end-customers only if it connects some of the content ISPs.

Theorem *1 (Shapley Value for Inelastic Demand):* Given a set of ISPs $\mathcal{C}\cup\mathcal{B}$ with topology $E \subseteq \tilde{E}$, the Shapley value of any ISP $B_i \in \mathcal{B}$ or $C_i \in \mathcal{C}$ under the inelastic demand is the following:

$$\varphi_{B_j} = \frac{d_{B_j}}{d_{B_j}+1}\alpha x_j + \sum_{q=1}^{|\mathcal{Q}|}\frac{d^q_{B_j}}{d^q_{B_j}+1}\beta k_q x_j,$$

$$\varphi_{C_i} = \sum_{\{j:(C_i,B_j)\in E\}}\Big[\frac{\alpha x_j}{(d_{B_j}+1)d_{B_j}} + \sum_{q\in Q_i}\frac{\beta k_q x_j}{(d^q_{B_j}+1)d^q_{B_j}}\Big].$$

Theorem 1 shows that under the inelastic demand, the Shapley values of the ISPs can be decomposed linearly as a function of customer sizes $\{x_j\}$. Each eyeball ISP B_j's Shapley value is proportional to its own customer size x_j, and is independent of the customer sizes of other eyeball ISPs. Further, this Shapley value φ_{B_j} can be decomposed as two parts: a fraction $d_{B_j}/(d_{B_j}+1)$ of the eyeball-side revenue αx_j and fractions $d^q_{B_j}/(d^q_{B_j}+1)$ of the content-side revenue $\beta k_q x_j$ generated by each content q. Consequently, content ISPs collect the remaining revenue. All content ISPs that connect to B_j evenly share $1/(d^q_{B_j}+1)$ of eyeball-side revenue αx_j. The $1/(d^q_{B_j}+1)$ of the content-side revenue $\beta k_q x_j$ is also evenly shared by the subset of content ISPs which provide content q.

We define $t_{ij}, i \in \mathcal{C}, j \in \mathcal{B}$ as a bilateral payment from ISP i to ISP j. The following corollary gives the closed-form bilateral payments that implement the Shapley value revenue distribution.

Corollary *1 (Bilateral Payments):* The bilateral payments between any linked pair of ISPs, i.e. $(C_i, B_j) \in E$, that implement the Shapley value are the following:

$$t_{B_jC_i} = \frac{\alpha x_j}{(d_{B_j}+1)d_{B_j}} = \frac{P_{B_j}}{(d_{B_j}+1)d_{B_j}},$$

$$t_{C_iB_j} = \sum_{q\in Q_i} \frac{\beta k_q x_j}{d^q_{B_j}+1} = \sum_{q\in Q_i} \frac{d^q_{B_j}}{d^q_{B_j}+1} P^{B_j,q}_{C_i}.$$

Corollary 1 implements the Shapley value revenue for ISPs using bilateral payments. Each payment can be expressed as fraction of the direct payment (P_{C_i} for a content ISP and P_{B_j} for an eyeball ISP) of an ISP. Each eyeball ISP B_j transfers $1/(d_{B_j}+1)$ of its direct payment P_{B_j} to connected content ISPs. $P^{B_j,q}_{C_i}$ denotes the fraction of direct payment P_{C_i} generated by B_j requesting content q. Therefore, each content ISP C_i, however, only keeps $1/(d^q_{B_j}+1)$ of its direct payment $P^{B_j,q}_{C_i}$ for every content q provided to B_j.

Corollary *2 (Marginal Revenue):* Suppose all content ISPs provide a set $\mathcal{Q}$ of contents. Let $K = \sum_{q\in\mathcal{Q}} k_q$. Consider any de-peering of a pair of ISPs, i.e. removing $(C_i, B_j) \in E$ from E to form $E^{'}$. The marginal revenues of the ISPs, defined as $\Delta_i = \varphi_i(E^{'}) - \varphi_i(E)$, are the following:

$$\Delta_{\varphi_{B_j}} = \Delta_{\varphi_{C_i}} = -\frac{(\alpha + K\beta)x_j}{d_{B_j}(d_{B_j}+1)} = -\frac{1}{d^2_{B_j}}\varphi_{B_j}(E),$$

$$\Delta_{\varphi_{C_l}} = \frac{2(\alpha + K\beta)x_j}{(d^2_{B_j}-1)d_{B_j}} \quad \forall \quad C_l : (C_l, B_j) \in E',$$

$$\Delta_{\varphi_{C_l}} = 0 \;\forall\; C_l : (C_l, B_j) \notin E, \quad \Delta_{\varphi_{B_l}} = 0 \;\forall\; B_l \neq B_j.$$

Corollary 2 shows the revenue effect on the pair of de-peering ISPs as well as other ISPs. The marginal revenue loss of a de-peering eyeball ISP B_j is inversely proportional to the square of d_{B_j}, which is the degree of connectivity to the content ISPs. For example, if B_j only connects to one content ISP, the marginal revenue is $-\varphi_{B_j}(E)$, which means that when the link is disconnected, the revenue loss is 100% of original Shapley revenue. Similarly, it loses $1/n^2$ of it Shapley revenue if it disconnects one of its n links. This result implies that when an eyeball, controlling inelastic customer demand, connects to more content ISPs, its marginal loss by disconnecting any of the content ISPs decreases inversely proportional to the degree of connectivity.

V. Elastic Customer Demand

In this section, we consider an elastic customer demand model. We assume that the total population size of end-customer is x. Consider any coalition $\mathcal{S} \subset \mathcal{N}$. Let $\mathcal{S} = \mathcal{C}_\mathcal{S} \cup \mathcal{B}_\mathcal{S}$ for some $\mathcal{C}_\mathcal{S} \subseteq \mathcal{C}$ and $\mathcal{B}_\mathcal{S} \subseteq \mathcal{B}$. We define the complete bipartite graph of the coalition $\mathcal{S}$ as $\tilde{E_\mathcal{S}} = \{(C_i, B_j) : C_i \in \mathcal{C}_\mathcal{S}, B_j \in \mathcal{B}_\mathcal{S}\}$. We further assume that if the topology of system is $\tilde{E_\mathcal{S}}$ for some $\mathcal{S} \subseteq \mathcal{N}$, the customer size of an eyeball ISP B_j is the following:

$$x_j = \begin{cases} \frac{x}{|\mathcal{B}_\mathcal{S}|} & \text{if } B_j \in \mathcal{B}_\mathcal{S}, \\ 0 & \text{otherwise.} \end{cases}$$

This elastic demand assumption implies that when some eyeball ISPs leave the system, their customers are re-distributed evenly to the remaining eyeball ISPs. It models a perfect elastic demand where users can choose any of the eyeball ISPs with equal probability. We do not put any assumption on the customer re-distribution when eyeball ISPs disconnect individual links to content ISPs.

Similarly to the definition in Equation 4, we denote d_q as the number of content ISPs in the coalition $\mathcal{C}_\mathcal{S}$ that provide content q, defined as $d_q = \sum_{i=1}^{|\mathcal{C}_\mathcal{S}|} \mathbf{1}_{\{q\in Q_i\}}$.

Theorem *2 (Shapley Value for Elastic Demand):* Given a subset of ISPs $\mathcal{C}_\mathcal{S} \cup \mathcal{B}_\mathcal{S}$ with a fully-connected topology $\tilde{E_\mathcal{S}} \subseteq \tilde{E}$, the Shapley value of any ISP $B_i \in \mathcal{B}_\mathcal{S}$ or $C_i \in \mathcal{C}_\mathcal{S}$ under the elastic demand is the following:

$$\varphi_{B_j} = \frac{|\mathcal{C}_\mathcal{S}|}{|\mathcal{B}_\mathcal{S}|(|\mathcal{C}_\mathcal{S}| + |\mathcal{B}_\mathcal{S}|)}\alpha x + \sum_{q=1}^{|\mathcal{Q}|} \frac{d_q}{|\mathcal{B}_\mathcal{S}|(d_q + |\mathcal{B}_\mathcal{S}|)}\beta x k_q,$$

$$\varphi_{C_i} = \frac{|\mathcal{B}_\mathcal{S}|}{|\mathcal{C}_\mathcal{S}|(|\mathcal{C}_\mathcal{S}| + |\mathcal{B}_\mathcal{S}|)}\alpha x + \sum_{q\in Q_i} \frac{|\mathcal{B}_\mathcal{S}|}{d_q(d_q + |\mathcal{B}_\mathcal{S}|)}\beta k_q x.$$

Theorem 2 shows the Shapley value of each ISP under an elastic demand model. These Shapley values are proportional to the total customer size x. The eyeball side revenue αx is shared by the coalition $\mathcal{S} = \mathcal{C}_\mathcal{S} \cup \mathcal{B}_\mathcal{S}$ and each content side revenue $\beta k_q x$ is share by $\mathcal{B}_\mathcal{S}$ and the set of content ISPs that provide content q. Since the demand is elastic, the group of B_S can be imagined as a whole to compete the revenue and divide revenue evenly among themselves. In particular, if $|\mathcal{B}_\mathcal{S}| = 1$, the results coincide with Theorem 1, because customers cannot switch to another eyeball ISP (inelastic demand) when there is only one eyeball ISP.

Corollary *3 (Bilateral Payments):* The bilateral payments between any linked pair of ISPs, i.e. $(C_i, B_j) \in \tilde{E_\mathcal{S}}$, that implement the Shapley value are the following:

$$t_{B_jC_i} = \frac{\alpha x}{|\mathcal{C}_\mathcal{S}|(|\mathcal{C}_\mathcal{S}| + |\mathcal{B}_\mathcal{S}|)} = \frac{|\mathcal{B}_\mathcal{S}|P_{B_j}}{|\mathcal{C}_\mathcal{S}|(|\mathcal{C}_\mathcal{S}| + |\mathcal{B}_\mathcal{S}|)},$$

$$t_{C_iB_j} = \sum_{q\in Q_i} \frac{\beta k_q x}{|\mathcal{B}_\mathcal{S}|(d_q + |\mathcal{B}_\mathcal{S}|)} = \sum_{q\in Q_i} \frac{d_q P^q_{C_i}}{|\mathcal{B}_\mathcal{S}|(d_q + |\mathcal{B}_\mathcal{S}|)}.$$

The bilateral payment from an eyeball ISP B_j is a fraction of the eyeball side revenue αx and an adjustment of its P_{B_j}. The eyeball ISP pays $|\mathcal{B}_\mathcal{S}|/(|\mathcal{C}_\mathcal{S}| + |\mathcal{B}_\mathcal{S}|)$ of its direct payment P_{B_j} to all content ISPs. Similarly, a content ISP C_i pays fractions of the content side revenue $\beta k_q x$, which are adjustments of their direct payment $P^q_{C_i}$ generated by content q.

Corollary *4 (Marginal Revenue):* Suppose all content ISPs provide a set $\mathcal{Q}$ of contents. Consider any de-peering of a content ISP with all eyeball ISPs, i.e. removing some $C_i \in \mathcal{C}_\mathcal{S}$ from $\mathcal{C}_\mathcal{S}$. Let the new set of ISPs to be $S' = S - \{C_i\}$. The marginal aggregate revenue for eyeball ISPs and content ISPs are defined as

$$\Delta_{\varphi_\mathcal{B}} = \varphi_\mathcal{B}(\mathcal{S}') - \varphi_\mathcal{B}(\mathcal{S}) = \sum_{B_j\in\mathcal{S}'} \varphi_{B_j}(\mathcal{S}') - \sum_{B_j\in\mathcal{S}} \varphi_{B_j}(\mathcal{S})$$

and

$$\Delta_{\varphi_\mathcal{C}} = \varphi_\mathcal{C}(\mathcal{S}') - \varphi_\mathcal{C}(\mathcal{S}) = \sum_{C_i\in\mathcal{S}'} \varphi_{C_i}(\mathcal{S}') - \sum_{C_i\in\mathcal{S}} \varphi_{C_i}(\mathcal{S})$$

respectively. These marginal aggregate revenue satisfies:

$$\Delta_{\varphi_\mathcal{B}} = -\Delta_{\varphi_{C_j}} = -\frac{|\mathcal{B}_\mathcal{S}|}{|\mathcal{C}_\mathcal{S}|(|\mathcal{C}_\mathcal{S}| + |\mathcal{B}_\mathcal{S}| - 1)}\varphi_\mathcal{B}(\mathcal{S}).$$

Corollary 4 shows the marginal revenue loss for the set of eyeball ISPs by losing one of content ISPs. Because the demand is elastic and customers can choose among eyeball ISPs, we consider the set of eyeball ISPs as a group, competing for the revenue with the set of content ISPs. This result implies that given less number of content ISPs, the marginal revenue loss is larger for all eyeball ISPs. Since the total revenue is fixed, the revenue loss of eyeball ISPs is redistributed to the remaining content ISPs. This result also generalizes Corollary 2, which is the case for $|\mathcal{B}_\mathcal{S}| = 1$.

VI. Examples

In our first example, we focus identical content ISPs that provide the same set of contents $\mathcal{Q}$. We compare the revenue obtained by the group of eyeball ISPs $\varphi_\mathcal{B} = \sum_{B_j\in\mathcal{B}_\mathcal{S}} \varphi_{B_j}$ and by the group of content ISPs $\varphi_\mathcal{C} = \sum_{C_i\in\mathcal{C}} \varphi_{C_i}$. Figure 4 illustrates the Shapley revenue distribution between eyeball ISPs and content ISPs under complete bipartite topology. On the x-axis and y-axis, we vary the number of eyeball and content ISPs. We plot the proportion of total revenue, i.e. $(\alpha + \beta\sum_{q\in\mathcal{Q}} k_q)x$, obtained by content ISPs on the z-axis. We can observe that the revenue for both groups of ISPs follows $\varphi_\mathcal{B} : \varphi_\mathcal{C} = |\mathcal{C}_\mathcal{S}| : |\mathcal{B}_\mathcal{S}|$. In particular, both types of ISPs evenly share revenue when $|\mathcal{B}_\mathcal{S}| = |\mathcal{C}_\mathcal{S}|$. For elastic customer demand, eyeball ISPs and content ISPs have anti-symmetric roles in the Shapley revenue distribution, i.e., a content ISP's revenue when there are x eyeball ISPs and y content ISPs equals what an eyeball ISP's revenue is when there are y eyeball ISPs and x content ISPs.

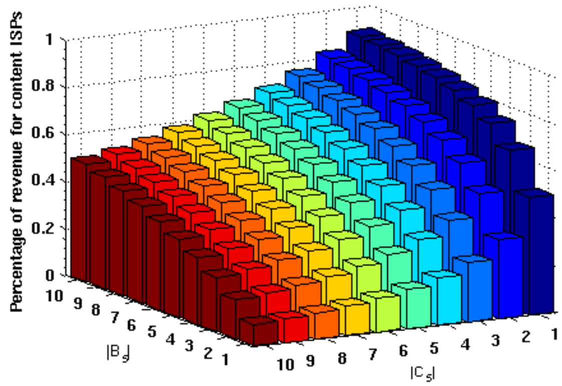

Fig. 4. Shapley revenue distribution between eyeball and content ISPs.

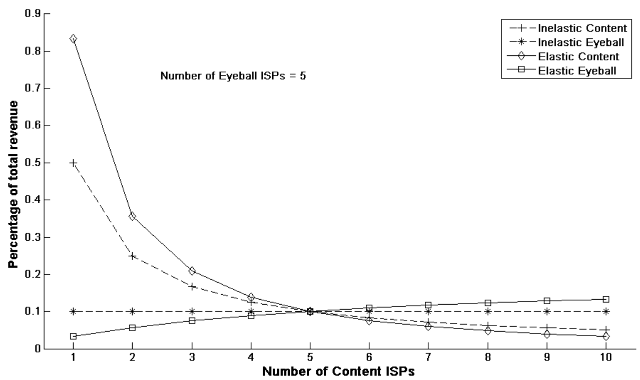

Fig. 5. Shapley revenue distribution for each eyeball/content ISP with inelastic/elastic demands.

Figure 5 compares revenues across elastic and inelastic settings. The number of eyeball ISPs is fixed at 5, and the number of content ISPs is varied on the x-axis. Individual ISP revenues for the two cases (elastic and inelastic customers) are plotted along the y-axis for both the content ISP and the eyeball ISP. The figure supports several interesting observations:

- When $|C_S| = |B_S|$, the symmetry desribed above results in content an eyeball ISPs evenly splitting revenue. This situation holds for both the case of elastic and inelastic customers.
- When $|C_S| < |B_S|$, content ISP revenues are larger when eyeball customers are elastic in comparison to when customers are inelastic. The reverse is true for eyeball ISP revenues.
- When $|C_S| > |B_S|$, the situation reverses, with content ISP revenues being larger when customers are inelastic than when elastic.

The above observations have some interesting implications. In an environment where the content market is dominated by a small set of players, and eyeball ISPs are numerous, eyeballs benefit from inelasticity, i.e., they should discourage customers from being able to move easily from one eyeball to another, suggesting that eyeball ISPs are better off monopolizing customers in regions. In contrast, if the content market has many more ISPs than the eyeball market, eyeball ISPs can increase market share by facilitating customer movement between them, e.g., ISPs share coverage of regions.

VII. Conclusion

In this paper, we explore ISP peering settlements in the context of sharing revenue among eyeball and content ISPs. Our solution is based on the Shapley value concept which provides various fairness and incentives to the ISPs. Our results show that the Shapley value revenue distribution can be implemented by bilateral payment between any pair of eyeball and content ISPs. Our results reveal that 1) under inelastic customer demand, the marginal revenue loss of an eyeball ISP from de-peering to a content ISP is inversely proportional to the square of number of connected content ISPs, and 2) under inelastic customer demand with complete bipartite topology, the revenue ratio between the groups of eyeball and content ISPs is inverse to ratio of number of ISPs in each group. Comparing with revenue under inelastic and elastic customer demand, we observe the conditions where eyeball ISPs can better off by monopolizing small regions or sharing coverage of regions. In practice, this bilateral implementation of the Shapley value gives a guideline for ISPs to negotiate partial peering agreements and for regulatory institutions to impose pricing regulations.

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APPENDIX

Before we give the proofs of the theorems, we state two more properties [11] of the Shapley value, which we are going to use in the proofs.

Property *1 (Balanced Contribution):* For any $i, j \in \mathcal{N}$, j's contribution to i equals i's contribution to j, i.e. $\varphi_i(\mathcal{N}, v) - \varphi_i(\mathcal{N} \backslash \{j\}, v) = \varphi_j(\mathcal{N}, v) - \varphi_j(\mathcal{N} \backslash \{i\}, v)$.

Property *2 (Additivity):* Given any two systems $(\mathcal{N}, v)$ and $(\mathcal{N}, w)$, if $(\mathcal{N}, v + w)$ is the system where the worth function is defined by $(v + w)(\mathcal{S}) = v(\mathcal{S}) + w(\mathcal{S})$, then $\varphi_i(\mathcal{N}, v + w) = \varphi_i(\mathcal{N}, v) + \varphi_i(\mathcal{N}, w)$ for all $i \in \mathcal{N}$.

Here we give the sketch proofs of Theorem 1 and 2. The proofs of the corollaries follow easily.

Proof of Theorem 1: We start with one eyeball ISP B with customer size x, connecting with n content ISPs. Each content ISP provides a single content with popularity factor k. We denote $\varphi_B(n)$ as the Shapley value revenue for the eyeball ISP. Similarly, we denote $\varphi_C(n)$ as the Shapley value revenue for each connected content ISP. Since we know the total revenue generated by all ISPs is $(\alpha + \beta k)x$, the following equation holds.

$$\varphi_B(n) + n\varphi_C(n) = (\alpha + \beta k)x \quad \forall \quad n = 1, 2, \cdots . \tag{8}$$

By the balanced contribution property of the Shapley value [11], we have the following equation.

$$\varphi_C(n) - 0 = \varphi_B(n) - \varphi_B(n-1) \quad \forall \quad n = 1, 2, \cdots . \tag{9}$$

Now we use proof by induction to show

$$\begin{cases} \varphi_B(n) = \frac{n}{n+1}(\alpha + \beta k)x, \\ \varphi_C(n) = \frac{1}{n(n+1)}(\alpha + \beta k)x. \end{cases}$$

When $n = 1$, $\varphi_B(1) = \varphi_C(1) = \frac{1}{2}(\alpha + \beta k)x$ which satisfies the above equations. Suppose the above equations hold for $n = m - 1$ for some m. Then for $n = m$, from Equation 9 we have

$$\varphi_C(m) = \varphi_B(m) - \varphi_B(m-1) = \varphi_B(m) - \frac{m-1}{m}(\alpha + \beta k)x.$$

From Equation 8, we have

$$\varphi_B(m) + m\varphi_C(m) = (\alpha + \beta k)x.$$

From the above two questions, we solve $\varphi_B(m)$ and $\varphi_C(m)$ as

$$\begin{cases} \varphi_B(m) = \frac{m}{m+1}(\alpha + \beta k)x, \\ \varphi_C(m) = \frac{1}{m(m+1)}(\alpha + \beta k)x. \end{cases}$$

Finally, we can have different sets of contents shared by content ISPs and multiple eyeball ISPs. Since the *worth function* v defined in Equation 1 is additive for different contents and different customer sizes $\{x_j\}$, by the additivity property of Shapley value, we conclude the result of Theorem 1 ■

Proof of Theorem 2: Similarly, we start from a single content with popularity factor k. The total population size is x. We define $\varphi_B(|\mathcal{C}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|)$ and $\varphi_C(|\mathcal{C}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|)$ to be the Shapley value for each eyeball and content ISP respectively. Suppose $|\mathcal{C}_\mathcal{S}| = m$ and $|\mathcal{B}_\mathcal{S}| = n$. In this case, since the total revenue generated by all ISPs is $(\alpha + \beta k)x$, we have the following equation

$$m\varphi_C(m, n) + n\varphi_B(m, n) = (\alpha + \beta k)x \quad \forall \quad m, n = 1, 2, \cdots . \tag{10}$$

By the balanced contribution property of the Shapley value, the follow equation holds for all $m, n = 1, 2, \cdots$.

$$\varphi_B(m,n) - \varphi_B(m-1,n) = \varphi_C(m,n) - \varphi_C(m,n-1). \tag{11}$$

We want to prove

$$\begin{cases} \varphi_B(|\mathcal{C}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|) = \frac{|\mathcal{C}_\mathcal{S}|}{|\mathcal{B}_\mathcal{S}|(|\mathcal{C}_\mathcal{S}|+|\mathcal{B}_\mathcal{S}|)}(\alpha+\beta k)x, \\ \varphi_C(|\mathcal{C}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|) = \frac{|\mathcal{B}_\mathcal{S}|}{|\mathcal{C}_\mathcal{S}|(|\mathcal{C}_\mathcal{S}|+|\mathcal{B}_\mathcal{S}|)}(\alpha+\beta k)x. \end{cases}$$

For the boundary condition $|\mathcal{C}_\mathcal{S}| = |\mathcal{B}_\mathcal{S}| = 1$, $\varphi_B(1,1) = \varphi_C(1,1) = \frac{1}{2}(\alpha+\beta k)x$, which satisfies the above equations. We use proof by induction. Suppose the above equations satisfy for $(|\mathcal{C}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|) = (m-1, n)$ and $(|\mathcal{C}_\mathcal{S}|, |\mathcal{B}_\mathcal{S}|) = (m, n-1)$. The balanced property equation becomes:

$$\varphi_B(m,n) - \frac{(m-1)(\alpha+\beta k)x}{n(m-1+n)} = \varphi_C(m,n) - \frac{(n-1)(\alpha+\beta k)x}{m(m+n-1)}.$$

$$\varphi_B(m,n) - \varphi_C(m,n) = \frac{(m-n)(\alpha+\beta k)x}{mn}.$$

Putting the above equation together with Equation 10, we obtain

$$\begin{cases} \varphi_B(m,n) = \frac{m}{n(m+n)}(\alpha+\beta k)x, \\ \varphi_C(m,n) = \frac{n}{m(m+n)}(\alpha+\beta k)x. \end{cases}$$

Finally, we can extend the above results for multiple contents using the additivity property of the Shapley value to reach Theorem 2. ∎