A Budget-balanced and Price-adaptive Credit Protocol for MANETs

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***Abstract*— A virtual credit exchange protocol for Mobile Ad-hoc Networks (MANETs) is proposed to enforce the cooperation of packet forwarding. In this protocol, each node decides a price (i.e. the number of credits) charged for forwarding a unit amount of traffic (i.e. one data packet). One node's pricing decision only depends on the prices of nodes that are going to forward packets for it. Each node updates its price adaptively and distributively. Under certain condition of the demand relationship between all nodes, this protocol converges to a unique price solution and provides sustainable and stable credit levels for all nodes. Experiments are performed to show that rational users have incentive to follow this protocol: 1) Setting higher prices do not help a node obtain more service. 2) Dropping other nodes' packets reduces a node's own throughput.**

I. INTRODUCTION

Mobile ad-hoc networks (MANETs) and wireless mesh networks (WMNs) have recently attracted much attention by their potential applications [15], [18], [19], [1]. With the rapid development of various wireless hardware devices and communication protocols (e.g. IEEE 802.11), the implementation of MANETs and WMNs become feasible.

Unlike traditional wired hosts, wireless nodes in ad-hoc networks are supposed not only to participate in the networks for their own services, but also to forward packets for other nodes voluntarily. This assumption is crucial for ad-hoc networks to work properly. If all nodes come from a single authority and work for the same objective, they have incentives to cooperate with each other. However, nodes are generally autonomous and naturally prone to refuse to forward packets for other nodes. Because forwarding packets may consume costly resources like battery power and CPU cycles[1].

In order to encourage autonomous nodes to forward packets for other nodes, incentive protocols for ad-hoc networks are needed. However, designing incentive protocols for ad-hoc networks has two folded challenges.

A. Theoretical and Practical Challenges

One of the two main streams of incentive work is credit based system [9], [21], [6], [5]. [2] The fundamental concept is that nodes have incentives to provide services (e.g. forward packets) to other nodes because they need the services provided by those nodes at a later time. During these service exchange processes, credit plays a role as exchange medium. For instance, nodes receive credits by forwarding packets for other nodes and pay credits by requesting such a service. One technical difficulty mentioned in [9] is the credit level maintained by the system. If the credit level is too high, each node has many credits to use and does not have incentive to provide service in order to receive more credits. If the credit level is too low, some nodes may not be able to obtain enough credit so as to get its own packets relayed. Periodically reset the system's credit level does not solve the problem. Because nodes do not have incentives to provide services before the credit level resets, which brings new credits to them. Some systems propose using monetary credits [21]. But real money brings extra burden to the system. It requires either a tamper proof hardware at each node or a centralized authority. The former is difficult to achieve [3] while the latter would weaken the self-organizing and decentralized nature of ad-hoc networks. Lastly, pricing the service (in terms of number of credits) is never trivial.

From a practical perspective of view, incorporating incentive into ad-hoc networks causes more challenges. First, an incentive protocol should be easy to deploy. It needs to be embedded in or built on top of current wireless ad-hoc protocols. This also implies that an incentive protocol must be distributed in nature. Second, an incentive protocol should also be incentive-compatible. This means that individual nodes will be willing to follow the protocol for their own sakes. This guarantees that each autonomous and self-organizing node, when given the choice to use or not use the protocol, will follow the protocol voluntarily.

B. Our Approaches and Contributions

In this work, we propose a *budget-balanced* credit based incentive system for ad-hoc networks. From an autonomous node's view, we consider the following question: **How many credits should be charged for a unit amount of traffic routed through?** Our approach is different from all existing work in the way that we make a separation of fairness and incentive. Traditional approaches [5], [6], [21] often assume either of the following: (1) Each node pays the same if they obtain the same amount of services. (2) Each node obtains the same number of credits if they provide the same amount of

[1]Battery power is limited for most mobile nodes. For wireless sensor nodes, processing power is limited and becomes a scarce resource.

[2]The other one is trust management system or reputation system. We will discuss it in related work.

workload. Due to the imbalance of the traffic demands between nodes, neither of these two approaches can sustain a stable credit level for each node. In our approach, although each node pays and gains proportionally to the service obtained and the workload done, each node decides its own price to charge for a unit traffic going through it. As a result, each node can sustain a credit level to afford its own traffic demand.

The choice of virtual credit in stead of monetary credit comes naturally because we do not want to sacrifice the self-organizing and self-contain system structure of ad-hoc networks. The "budget-balance" feature of the protocol has two meanings. First, when a serving node help a requesting node forward packets, the number of credits paid from the requesting node equals the number of credits received by the serving node. Second, with a fixed number of nodes, the total number of virtual credits in the network is a constant. Thus, one can imagine that the flow of data packets should be in the same direction of the credit flow.

Our credit exchange protocol has three advantages. First, the protocol is incentive-compatible. This means that each node has an incentive to follow the protocol. As we will see later, if any node refuses to forward packets for other node at some time, it will not obtain enough credit for its own packets got through the network. Second, each node decide how many credits it needs to charge distributively and adaptively. Third, by following the protocol, not only the system has a stable credit level (which is a constant), but individual nodes have stable credit levels as well.

II. Virtual Credit Exchange

In this section, we consider a set of nodes that obtain forwarding services by paying virtual credits to other nodes. We start from a motivating example to show what the price solutions, which sustain the forwarding demands as well as stabilize credits, are. Next, we formalize a pricing problem which our credit exchange protocol solves. After characterizing the price solutions from a global perspective, we finally describe how the credit exchange protocol distributively and adaptively obtains them.

A. A Motivating Example

Suppose we have two nodes that need each other's help for forwarding packets. The demand rate of node j from node i is defined as d_{ji}, which also means that node i wants node j to forward its traffic at a rate of d_{ji} amount of data (e.g. number of packets) per time unit (e.g. one second).

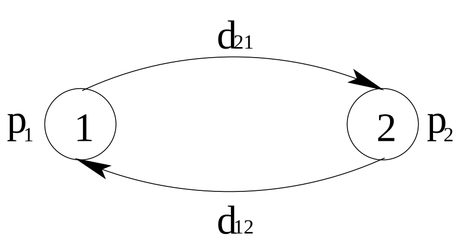

Fig. 1. A two-node example

Figure 1 shows the traffic directions and the according demand rates. We define p_i as the price of node i, which mean the number of credit charged by node i for forwarding a packet. We can imagine that node i stores p_j credits in its packet and send it to node j. Node j obtains the credits from the packet and forward it to the next hop (or destination). Other implementation of the credit exchange may also be possible, for example, the credits can be stored in the acknowledgement packets from the destinations. Consequently, node j pays $p_i d_{ij}$ number of credits to node i every second. Suppose each node has a limited number of credits. In order to have enough credits for both nodes to circulate for a long time, the credit flows have to be balanced:

$$p_1 d_{12} = p_2 d_{21} \quad \texttt{or} \quad \frac{p_1}{p_2} = \frac{d_{21}}{d_{12}}.$$

Before we reveal the full insights of price solutions, let us look at the above specific solution for a while. We observe two characteristics of the solution $\{p_1, p_2\}$:

1) The ratio $p_1 : p_2$ is a constant. This implies that the price solutions only depends on price ratio rather than the absolute value of the individual prices.
2) Prices are related to the workload done by nodes. In the above example, each node's price is proportional to its demand request and inverse proportional to its own workload.

B. The Pricing Problem

In a more general setting, we assume there are N nodes in the system. When a node joins the network at the first time, it receives the same amount of virtual credit from the system, say m credits. The total number of credits circulating in a system is proportional to the number of nodes in the system, i.e. Nm credits in total when there are N nodes in the system. Let us define the aggregate demand rate received by node i as

$$\bar{d}_i = \sum_{j=1}^{N} d_{ij}.$$

If node i forwards all the requested packets, $\bar{d}_i$ can be regarded as the workload node i has done in a unit time. Let us denote $\mathbf{p} = (p_1, p_2, \ldots, p_N)^T$ as the price vector of all nodes, where each p_i denotes the price of node i. We address the pricing problem to be: **Finding a price vector $\mathbf{p} > 0$ such that every node has enough credits to sustain its demand rates in a long run.**

From each node's view, for any fixed $\mathbf{p}$, it can afford its demand rates in a long run if and only if it earns at least as many credits as it consumes. Because the total number of credits is a constant, when a node's long run income of credits is larger than its expenditure, it absorbs credits. Eventually, other nodes will not have any credit to circulate. Therefore, in order to make all nodes' traffic rates affordable in a long run, each node's earning rate of credits must equal its expenditure rate. Mathematically, the pricing problem is to find a price vector $\mathbf{p} > \mathbf{0}$ such that the following balancing equations are satisfied.

$$p_i \bar{d}_i = \sum_{j=1}^{N} p_j d_{ji} \quad \forall i \in \mathcal{N}. \tag{1}$$

Let $D = \{d_{ij} : i, j \in \mathcal{N}\}$ be an $N \times N$ demand rate matrix. Let $\tilde{D} = \{\tilde{d}_{ij} : i, j \in \mathcal{N}\}$ as the rate balancing matrix, where $\tilde{d}_{ij} = -\bar{d}_i$ if $i = j$ and $\tilde{d}_{ij} = d_{ij}$ otherwise. The balancing equations (1) is equivalent to the following matrix form equation.

$$\mathbf{p}^T \tilde{D} = \mathbf{0}^T. \tag{2}$$

where

$$\tilde{D} = D - diag(\bar{\mathbf{d}}) = \begin{pmatrix} -\bar{d}_1 & d_{12} & \dots & d_{1N} \\ d_{21} & -\bar{d}_2 & \dots & d_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ d_{N1} & d_{N2} & \dots & -\bar{d}_N \end{pmatrix}$$

C. The Price Solutions and Characteristics

Obviously, if we allow $\mathbf{p} = \mathbf{0}$, it is a solution of the pricing problem. This solution means that nodes do not charge any credit for forwarding packets. Consequently, no enforcement is placed on nodes, and nodes have no incentives to forward packets.

Notice that the balancing matrix $\tilde{D}$ has two properties. First, every non-diagonal entry is non-negative, e.g. $\tilde{d}_{ij} \geq 0 \; \forall (i \neq j)$. Second, each row of $\tilde{D}$ has a zero sum. These qualifies $\tilde{D}$ to be a rate transition matrix for some continuous Markov chain. Therefore, we can characterize the solution $\mathbf{p}$ by exploiting the property of the conceptual Markov chain represented by $\tilde{D}$.

1) Irreducibility:

Theorem *1:* If the conceptual Markov chain represented by $\tilde{D}$ is irreducible, there exists a unique unit vector $\mathbf{v}$, such that any price solution $\mathbf{p} > 0$ is in the form of $\mathbf{p} = k\mathbf{v}$ for some $k > 0$.

The irreducibility of $\tilde{D}$ implies that each pair of nodes have direct or indirect forwarding demand on each other. The above theorem also generalizes the first observation in the motivating example, which shows that only price ratios determine the price solution.

2) Reducible Cases: In reality, $\tilde{D}$ may not be irreducible. Let us discuss the implications and the price solutions when $\tilde{D}$ is not irreducible.

In the first case, $\tilde{D}$ may consist several irreducible subgroups. Nodes within a subgroup can communicate with each other, but cannot communicate with nodes outside the subgroup. A typical example consists two irreducible subgroups. We can always relabel the nodes and construct the $\tilde{D}$ as follows.

$$\tilde{D} = \begin{pmatrix} D_1 & 0 \\ 0 & D_2 \end{pmatrix}$$

From the above $\tilde{D}$ matrix, we can see that nodes in D_1 cannot communicate with nodes in D_2. Physically, nodes between different subgroups have no forwarding demand on each other. Therefore, we can consider the pricing problems on each irreducible subgroup independently.

In the second case, the communication between some pairs (or subgroups) of nodes is uni-directional. A typical example consists two subgroup D_1 and D_2 where nodes in D_2 can communicate with nodes in D_1, but not vice versa. We can label all nodes in D_1 with the smallest indices:

$$\tilde{D} = \begin{pmatrix} D_1 & 0 \\ D_3 & D_2 \end{pmatrix}$$

From the above $\tilde{D}$ matrix, we find that nodes in D_1 has no transition to nodes in D_2, because the upper-right sub-matrix is a zero matrix. This implies that nodes in D_2 has no forwarding demand on nodes in D_1. Thus, no virtual credit will go to nodes in D_1 from nodes in D_2. However, since the matrix $D_3 > 0$, there is positive forwarding demand from D_1 to D_2. In order to have the virtual credits balanced in a steady state, all nodes in D_2 have to set their prices to be zero.

Again, it is undesirable to set a zero price for some node, because other nodes can use the service freely and have no incentive to provide their own services. But, a network may have a set of edge nodes which locate at the frontier of the network and need to send packets to other nodes. Because of the geographical locations of the edge nodes, no other node wants them to forward packets. We can imagine that these edge nodes can form a set D_1 in the above example. For mathematical tractability, we may need to set zero prices for nodes which are used by the edge nodes. However, this might be unfavorable in practice. We will address this problem as one of our future work.

3) Price Ratio and Workload: As seen in the the motivating example, the price ratio relates to the workload done by each node. Let us see how the workload affects the price ratios in the price solution using the following example. Suppose we have three nodes with two sets of demands and according price solutions shown in Figure 2. The demand of node 1 equals 1 and 3 accordingly in both scenarios. We normalize $p_2 = 1$ for the ease to compare both price solutions.

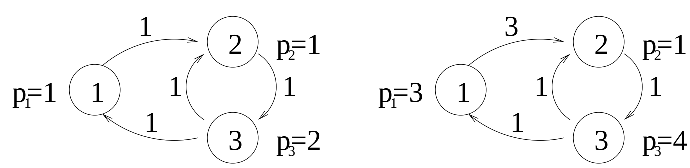

Fig. 2. price ratio and workload comparison

We observe that when node 1 increases its demand rate, the price ratio between node 2 and 3 changes from $p_2 : p_3 = 1 : 2$ to $1 : 4$. An intuitive reason is that node 2 forwards more traffic so that it has to charge relatively less to balance the credits. If a node i only request one node, say node j, to forward traffic (e.g. node 1 and 2 in the example), from the balancing equation we have

$$p_i \bar{d}_i = p_j d_{ji} \Longleftrightarrow \frac{\bar{d}_i}{d_{ji}} = \frac{p_j}{p_i}.$$

$\bar{d}_i$ can be regarded as the workload done by node i for other nodes to forward packets. d_{ji} can be regarded as the workload done for node i by other nodes. Thus, the price ratio equals the inverse of workload ratio. Generally, each node's workload equals (derived from Equation 1):

$$\bar{d}_i = \sum_{j=1}^{N} \frac{p_j}{p_i} d_{ji}.$$

We can think of one node's workload $\bar{d}_i$ as a weighted sum of the workload done by other nodes, where the weights are the price ratios.

D. Distributed Price Control and Convergence

Theoretically, we can obtain a price solution $\mathbf{p}$ by knowing the balancing matrix $\tilde{D}$. In practice, the global information $\tilde{D}$ may not be available to all nodes. We propose a distributed pricing algorithm for each node based on their local information. We prove that this distributed algorithm converges to a price solution $\mathbf{p}$ such that Equation (2) satisfies.

For each node, the essence of the price control algorithm is to set a price so that the node's earning of credits balances its expenditure. Let us denote $p_i(t)$ as the price of node i to forward a packet at time t. Similar to Equation (1), each node i updates its price as follows:

$$p_i(t+\delta t)\bar{d}_i = \sum_{j=1}^{N} p_j(t) d_{ji} = \sum_{j:d_{ji}>0} p_j(t) d_{ji} \qquad (3)$$

Operationally, the right hand side equals the rate of credits node i spent at time t. Node i adjust its price at time $t+\delta t$ such that its future earning rate equals the expenditure rate at time t. In a matrix form, the prices evolves as follows:

$$\mathbf{p}(\mathbf{t}+\delta\mathbf{t})^T diag(\bar{\mathbf{d}}) = \mathbf{p}(\mathbf{t})^T D$$

$$\Longleftrightarrow [\mathbf{p}(\mathbf{t}+\delta\mathbf{t})^T - \mathbf{p}(\mathbf{t})^T] diag(\bar{\mathbf{d}}) = \mathbf{p}(\mathbf{t})^T \tilde{D}$$

Suppose nodes can continuously change their prices. When δt tends to zero, the following autonomous system describes the evolution of the prices.

$$\dot{\mathbf{p}}^T diag(\bar{\mathbf{d}}) = \mathbf{p}^T \tilde{D} \qquad (4)$$

Theorem *2:* The autonomous system $\dot{\mathbf{p}}^T diag(\bar{\mathbf{d}}) = \mathbf{p}^T \tilde{D}$ converges to the solution set $\mathcal{M} = \{\mathbf{p} \mid \mathbf{p}^T \tilde{D} = \mathbf{0}^T\}$.

III. Incentive Properties and Behaviors

In the previous section, nodes decide their prices $\{p_i\}$ distributively and adaptively by equalizing the earnings and the expenditures in virtual credits. In this section, we exploit the effects of nodes' misbehavior, which may be conducted by nodes to improve their own performance. We show that misbehaviors can either be irrational for nodes to conduct or be harmless.

Throughout this section, we consider the following setting of an experiment. We consider five nodes. Each node initiates a traffic flow which needs to go through two of other nodes. These five flows are $1 \to 2 \to 3$, $2 \to 3 \to 4$, $3 \to 2 \to 5$, $4 \to 5 \to 1$ and $5 \to 1 \to 2$. The corresponding traffic demand rates are $200, 300, 250, 100$ and 150 Kbps. Initially, each node has 10000 virtual credits and sets its price to be 1 credit/Kb. Each node updates its price by every random period of time which is uniformly distributed with mean 10 seconds. Figure 3 shows the price, the number of credits and the average throughput of each node. We find that the price converges around 1 minute. After that, the number of credits for each node is also stabilized. Each node achieves the traffic rate they demand.

A. Price Effect

In Section II, we assume that each node i updates its price according to Equation (3). One intuitive misbehavior is that one node may set a higher price and stick to it, hoping that it can obtain more virtual credits and discourage traffic demands from other nodes (because they cannot afford it).

We assume that one of the nodes (i.e. node 4) doubles its price at time 200 (sec.) and keeps using this price afterward. Figure 4 shows that after node 4 doubles its price, all other nodes adaptively changes their prices and reach new stable prices. Although node 4 obtains more credits temporarily, all nodes achieve the traffic rate that they demand.

We find that during the price change, the node that increases price obtains more credits. Next, we assume that each node initially only have 1000 credits. Let us see how the price change affects the performance of the nodes. From Figure 5, we find that node 4 absorb all credits. But surprisingly, all other nodes' throughput are only affected temporarily. Because when all nodes reach a stable state, their credits are balanced. Therefore, although one node may not have many credits, it obtains the traffic rate it needs.

B. Dropping Effect

In our budget-balanced credit system, each node has the incentive to forward packets for other nodes. Only by forwarding other nodes' packets could one node obtain enough credits to sustain its own traffic demand rate.

In the following experiment, node 4 starts dropping half of the packets going through it at time 200 (sec.). From Figure 6, we find that node 2's throughput drops at time 200 (sec.). This is because node 4 is the last hop of its traffic. In return, node 2 starts accumulating node 4's credits at that time. After node 4 depletes all its credits, its throughput drops. Because it does not have enough credits to afford its original traffic rate. We conclude that rational nodes that want to obtain their traffic demand rates will not drop other nodes' packets.

IV. Related Work and Future Work

A. Related Work

Distributed and autonomous computer and communication systems need incentive mechanisms in nature, i.e. differentiated scheduling [11] and P2P file sharing systems [13]. In particular, mobile ad-hoc networks need an incentive mechanism to encourage node to forward data packets for other nodes. In

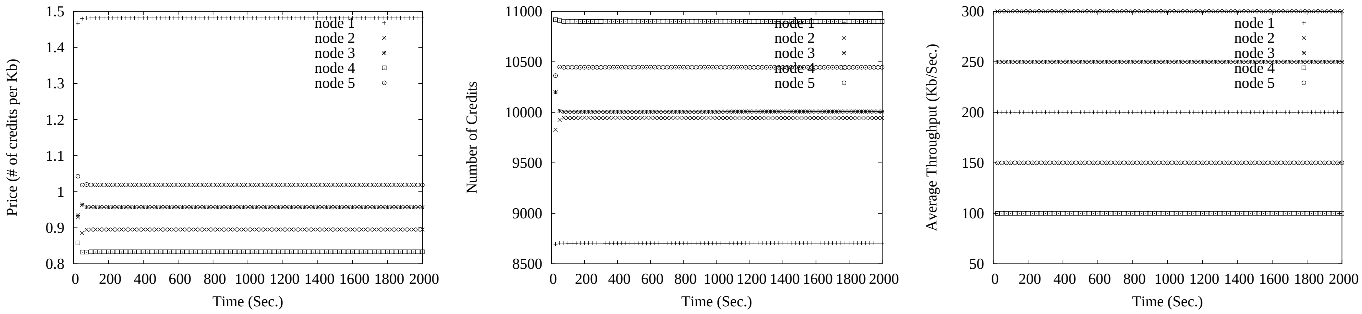

Fig. 3. Basic Setting: Price, Credits and Avg. Throughput

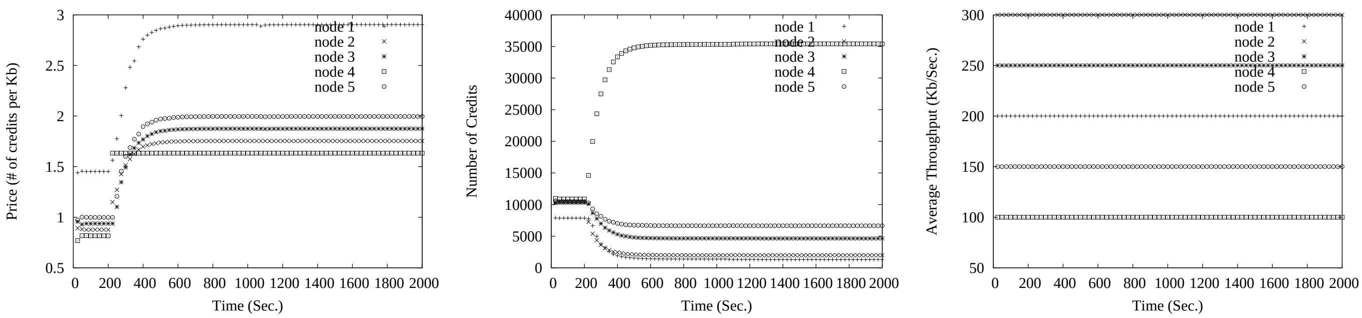

Fig. 4. Price Effect (Node 4 doubles price at time 200.)

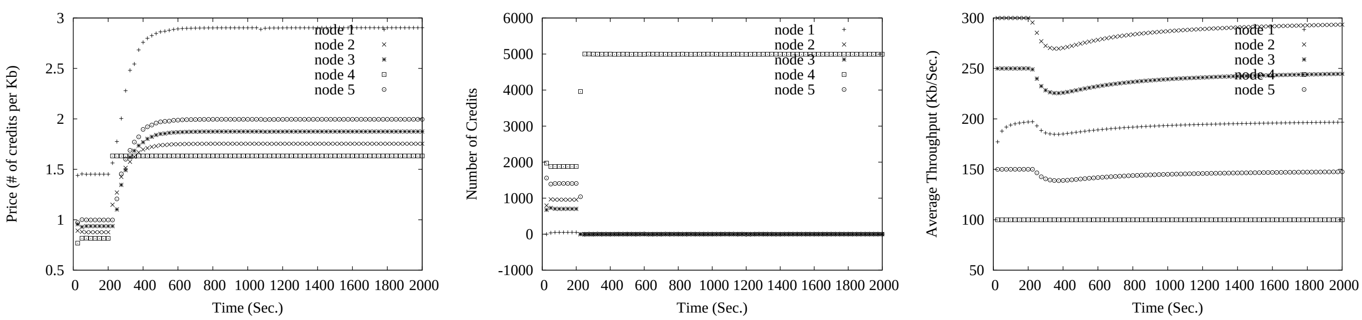

Fig. 5. Price Effect with Limited Credits

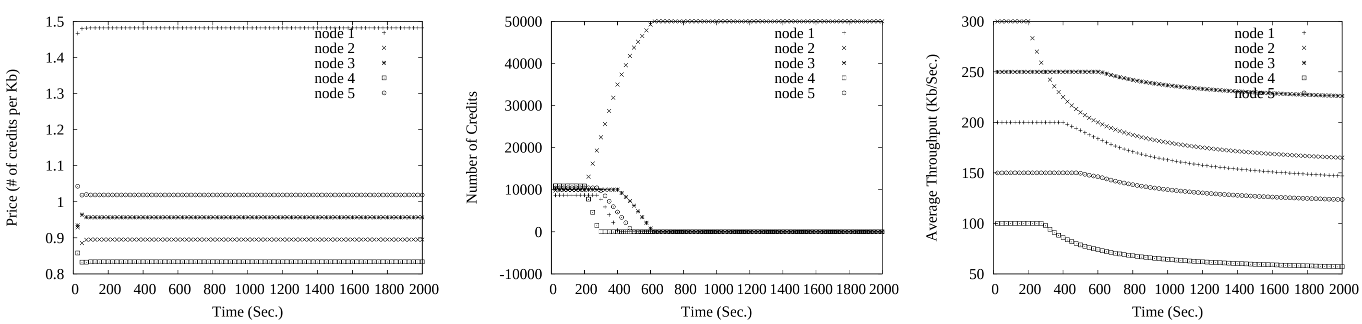

Fig. 6. Dropping Effect

general, an incentive mechanism encourages users to provide services by rewarding either monetary or non-monetary credits which they can use later in order to obtain services from the network.

For monetary mechanisms, the central problem is how to price the services provided by each node. As proposed in [7], multiple nodes, which provide the same service, compete for user demands in order to maximize profits. More delicate mechanisms use the idea of Vickrey-Clarke-Groves (VCG) mechanism [20], [2], where users bid for the services they want. VCG mechanism has the property that the best strategy for a user is to reveal the true value that they are willing to pay for the services.

One general discussion about incentive mechanisms for

MANETs can be found in [9], which categorizes non-monetary mechanisms into virtual credit systems (or token based systems) and reputation systems (trust management systems).

Proposed virtual credit mechanisms [21], [6], [5], [10], [17] are often locally budget-balanced, where one node receives the same amount of credits as the other node pays. But none of them considers a globally budget-balanced mechanism where total amount of credits in the system is stable. Thus, the credit levels of nodes may not be stable and nodes cannot obtain services when spending all virtual credits. In [10], the authors proposed a self-recharging mechanism. Consequently, nodes do not have incentives to provide services, because they can obtain new credits during the recharge processes.

Reputation systems also attract much attention in P2P systems [16], [4] as well as in wireless networks [14]. E-bay and Amazon are two real-world reputation systems where users choose their auctions based on the sellers' *reputation ratings*. Compared with virtual credit systems, reputation systems are more qualitative rather than quantitative. Although the reputation ratings are given by third-party users, reputation systems can also be vulnerable to false accusations or false praise.

B. Future Work

Theoretically, we will consider the case where the demand relationship is not irreducible. Physically, this happens to edge nodes, which do not have much traffic routed through them, therefore, do not have many incoming credits.

Practically, we will also consider how to securely implement the system so that nodes cannot cheat on their credits. How to use multi-path routing protocols to choose "affordable" routes for our virtual credit system is an another practical consideration.

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V. APPENDIX

Proof of Theorem 1: $\tilde{D}$ represents a continuous time Markov chain with finite state. Therefore, it is ergodic, and has a unique steady state distribution $\mathbf{v}$, which satisfies

$$\mathbf{v}^T \tilde{D} = \mathbf{0}^T.$$

Consequently, any $\mathbf{p} = k\mathbf{v}$ for some $k > 0$ satisfies

$$\mathbf{p}^T \tilde{D} = \mathbf{0}^T.$$

On the other hand, if there exist a price solution $\mathbf{p} = k\mathbf{w}$ for some $k > 0$ and some unit vector $\mathbf{w} \neq \mathbf{v}$, then $\mathbf{w}$ must also be a steady state distribution of the Markov chain. But the unique steady state distribution is $\mathbf{v}$. Here, we have a contradiction. ∎

Proof of Theorem 2: Let us define a Lyapunov function as follows:

$$V(\mathbf{p}) = \mathbf{p}^T \tilde{D} \tilde{D}^T \mathbf{p}.$$

Let us define $\mathcal{M} \equiv \{\mathbf{p} \mid \dot{V}(\mathbf{p}) = 0\}$. Because

$$\dot{V}(\mathbf{p}) = 2\tilde{D}^T \mathbf{p} = 0 \Longleftrightarrow \mathbf{p}^T \tilde{D} = \mathbf{0}^T$$

We know $\mathcal{M} \equiv \{\mathbf{p} \mid \dot{V}(\mathbf{p}) = 0\} = \{\mathbf{p} \mid \mathbf{p}^T \tilde{D} = \mathbf{0}^T\}$. Let us also define the vector $\hat{\mathbf{d}} = (1/\bar{d}_1, 1/\bar{d}_2, \ldots, 1/\bar{d}_N)$. The autonomous system is equivalent to:

$$\dot{\mathbf{p}}^T = \mathbf{p}^T \tilde{D}\ diag(\hat{\mathbf{d}})$$

Let us evaluate $\dot{V}(\mathbf{p})$ using the above equation.

$$\begin{aligned}\dot{V}(\mathbf{p}) &= \dot{\mathbf{p}}^T\tilde{D}\tilde{D}^T\mathbf{p} + \mathbf{p}^T\tilde{D}\tilde{D}^T\dot{\mathbf{p}} \\ &= \mathbf{p}^T\tilde{D}\ diag(\hat{\mathbf{d}})\tilde{D}\tilde{D}^T\mathbf{p} + \mathbf{p}^T\tilde{D}\tilde{D}^T\ diag(\hat{\mathbf{d}})\tilde{D}^T\mathbf{p} \\ &= \mathbf{p}^T\tilde{D}[diag(\hat{\mathbf{d}})\tilde{D} + \tilde{D}^T diag(\hat{\mathbf{d}})]\tilde{D}^T\mathbf{p} \\ &= \mathbf{p}^T\tilde{D}[diag(\hat{\mathbf{d}})D - I + D^T diag(\hat{\mathbf{d}}) - I]\tilde{D}^T\mathbf{p}\end{aligned}$$

Because $diag(\hat{\mathbf{d}})D$ can be regarded as a stochastic matrix whose row sum equals 1. By Perron-Frobenius[8] theorem, its all eigenvalues are less than or equal to 1. Thus, both $diag(\hat{\mathbf{d}})D - I$ and $D^T diag(\hat{\mathbf{d}}) - I$ has eigenvalues less than or equal to 0. As a result, the matrix $diag(\hat{\mathbf{d}})D - I + D^T diag(\hat{\mathbf{d}}) - I$ is semi-negative definite, and therefore, $\dot{V}(\mathbf{p}) \leq 0$.

Finally, by LeSalle's [12] theorem, every initial $\mathbf{p}$ converges to the set $\mathcal{M} \equiv \{\mathbf{p} \mid \dot{V}(\mathbf{p}) = 0\} = \{\mathbf{p} \mid \mathbf{p}^T\tilde{D} = \mathbf{0}^T\}$. ∎